P-wave AVOA for vertically fractured media

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ABSTRACT

We present a formula for the $R_{PP}-$ reflection coefficient for the case that the P-wave propagates in an isotropic halfspace and is reflected at the boundary of a vertically fractured halfspace.
The formula is derived by combining an approximate solution for the $R_{qPqP}-$ reflection coefficient with the results of an effective medium theory for representation of a cracked medium.

INTRODUCTION

A set of vertical fractures in an isotropic background medium causes anisotropy.
The resulting medium is of hexagonal type with a horizontal axis of symmetry ((Hudson, 1980), (Hudson, 1981)).
We connect the elastic moduli with an approximate solution for the $R_{qPqP}-$ reflection coefficient. The resulting formula is of the following form :

$$R_{PP}(\vartheta_p, \varphi, \epsilon, \sigma) = R_{PP}^{(iso)}(\vartheta_p) + R_{PP}^{(C)}(\vartheta_p, \varphi, \epsilon, \sigma). \tag{1}$$

Here ϑ_p is the angle of the phase normal of the incident wave with the vertical, φ is the azimuth angle of the receiver profile with the x-z-plane (the plane normal to the cracks), ϵ is the crack density and σ is the Poisson ratio of the isotropic background medium. In equation (1) the isotropic coefficient for a model without cracks is separated from the influence of the crack system.

THEORY

The elastic moduli for a system of vertically oriented dry cracks are given in first order in crack density ϵ by ((Hudson, 1980), (Hudson, 1981)) :

$$\overline{c}_{11} = -\epsilon \, \frac{16}{3}(\lambda + 2\mu)\frac{(1-\sigma)^2}{1-2\sigma}, \tag{2}$$

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$$
\begin{aligned}
\overline{c}_{12} &= -\epsilon\,\frac{16}{3}(\lambda+2\mu)\frac{\sigma(1-\sigma)}{1-2\sigma}, \\
\overline{c}_{22} &= -\epsilon\,\frac{16}{3}(\lambda+2\mu)\frac{\sigma^2}{1-2\sigma}, \\
\overline{c}_{55} &= -\epsilon\,\frac{8}{3}(\lambda+2\mu)\frac{1-2\sigma}{2-\sigma},
\end{aligned}
$$

$$
\overline{c}_{13}=\overline{c}_{12}, \quad \overline{c}_{33}=\overline{c}_{23}=\overline{c}_{22}, \quad \overline{c}_{66}=\overline{c}_{55}.
$$

λ, μ and σ are Lamé parameters and Poisson ratio of the isotropic background medium.

It is possible to show that the assumption of a weak contrast between the two elastic halfspaces leads to a simple expression for the $R_{qPqP}-$ reflection coefficient valid for general anisotropy (Zillmer, Gajewski & Kashtan).
The coefficient is given by

$$
\begin{aligned}
R_{qPqP} &= \frac{1}{4}\left(\frac{\Delta c_{33}}{\overline{\lambda}+2\overline{\mu}}+\frac{\Delta\rho}{\overline{\rho}}\right)-\frac{1}{4}\frac{\Delta\rho}{\overline{\rho}}\tan^2\vartheta_p \\
&+ \frac{1}{4}\frac{2\Delta c_{13}-\Delta c_{33}-4\Delta c_{55}}{\overline{\lambda}+2\overline{\mu}}\sin^2\vartheta_p+\frac{1}{4}\frac{\Delta c_{11}}{\overline{\lambda}+2\overline{\mu}}\sin^2\vartheta_p\tan^2\vartheta_p.
\end{aligned} \tag{3}
$$

Here Δ denotes the difference of the quantities between both halfspaces, $\overline{\lambda}, \overline{\mu}$ and $\overline{\rho}$ are Lamé parameters and density of an isotropic medium which serves as a reference model in the application of perturbation theory. Equation (3) is valid for two halfspaces of arbitrary anisotropy with the restrictions of weak anisotropy, a weak contrast in elastic parameters and for undercritical incidence. It represents a generalization of the formulas for isotropic ((Aki and Richards, 1980)) and transversely isotropic ((Thomsen, 1993)) halfspaces.

In the case of the fractured medium we get

$$
\begin{aligned}
R_{PP} &= R_{PP}^{(iso)} \\
&- \epsilon\,\frac{4}{3}\frac{\sigma^2}{1-2\sigma}\frac{1}{\cos^2\vartheta_p} \\
&+ \epsilon\,\frac{8}{3}\frac{1-4\sigma+\sigma^2}{2-\sigma}\cos^2\varphi\sin^2\vartheta_p \\
&- \epsilon\,\frac{4}{3}\frac{2(1-\sigma^2)-\sigma(1-2\sigma)\cos^2\varphi}{2-\sigma}\cos^2\varphi\sin^2\vartheta_p\tan^2\vartheta_p,
\end{aligned} \tag{4}
$$

with the isotropic part of the reflection coefficient given by (Aki & Richards, 1980) :

$$
R_{PP}^{(iso)}=\frac{1}{2}\Big(1-4\frac{\overline{v}_s^2}{\overline{v}_p^2}\sin^2\vartheta_p\Big)\frac{\Delta\rho}{\overline{\rho}}+\frac{1}{2\cos^2\vartheta_p}\frac{\Delta v_p}{\overline{v}_p}-4\frac{\overline{v}_s^2}{\overline{v}_p^2}\sin^2\vartheta_p\frac{\Delta v_s}{\overline{v}_s}. \tag{5}
$$

EXAMPLES

We discuss the model which was investigated by ((Strahilevitz and Gardner, 1995)) and by ((S. Mallik and Chambers, 1996)) : an isotropic Taylor shale with parameters

$v_p = 4.153\ km/s,\ v_s = 2.419\ km/s,\ \rho = 2.6\ g/cm^3$
and a fractured Austin chalk with parameters
$v_p = 4.969\ km/s,\ v_s = 2.615\ km/s,\ \rho = 2.57\ g/cm^3,\ \sigma \approx 0.31,\ \epsilon = 0.1.$
In case of dry cracks the AVO gradient changes the sign when measurements in the crack-strike plane and in the crack-normal plane are compared. The sign change does not occur in case of wet cracks. This can be explained with help of equation (3). The dominant term in this example is the term $\sim \Delta c_{11}$. For dry cracks Δc_{11} changes the sign when compared in both symmetry planes. The elastic parameter c_{11} for the isotropic Taylor shale is $44.84\ GPa$. For the fractured Austin chalk we have in case of dry cracks $57.64\ GPa$ in the crack-strike plane and $34.20\ GPa$ in the crack-normal plane, and in case of water-filled cracks $57.45 GPa$ and $62.26\ GPa$ respectively (Strahilevitz & Gardner, 1995). The result can also be explained with help of equation (4). For an incidence angle of $\vartheta = 45°$ in the crack-normal plane the third (anisotropic) term decreases the isotropic coefficient by approximately $\frac{2}{3}\epsilon$. This decrease is approximately twice as large as the decrease in the crack-strike plane (Figs. 1 and 2).

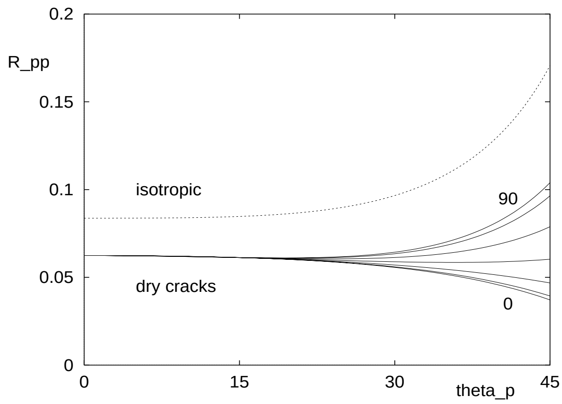

Figure 1: Exact $R_{PP}-$ coefficient for isotropic Taylor shale and fractured Austin chalk.

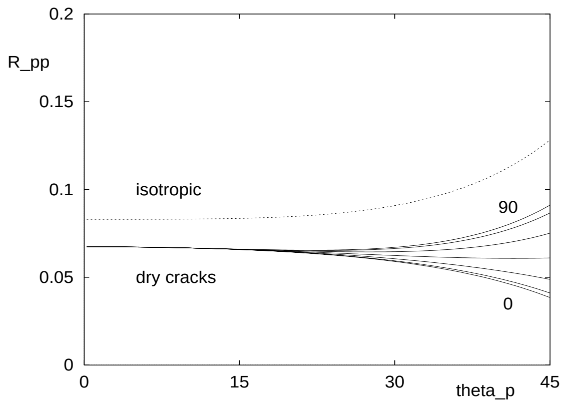

Figure 2: Approximate $R_{PP}-$ coefficient (eq.(3)) for the model of Fig.1.

DISCUSSION AND CONCLUSIONS

We derived explicit expressions for the $R_{PP}-$ reflection coefficient for a vertically fractured medium. The formulas show the influence of the Poisson ratio, the crack density and the azimuth of the receiver profile on the reflection coefficient. The formulas allow us to directly link parameters of the fractured rock to the AVOA response.

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