*Wave Inversion Technology, Report No. 1, pages 147-**??***

Frequency-dependent shear-wave splitting in finely layered media with intrinsic anisotropy

U. Werner and Sergei A. Shapiro[1]

keywords: *elastic wave theory, anisotropy, finely layered media, shear-waves*

ABSTRACT

Seismic wave propagation in 1-D randomly layered media is characterized by frequency-dependent anisotropy due to multiple scattering at 1-D inhomogeneities. An analytical description of the transmissivity and reflectivity in such media is given by the generalized O'Doherty-Anstey formulas, which are valid in the entire frequency domain and also for oblique incidence. They can be regarded as a combination and generalization of the Backus averaging ((Backus, 1962)), which is valid in the static limit only, and the theory of (O'Doherty and Anstey, 1971), which applies for all frequencies, yet for vertical incidence only. The transmission of obliquely incident plane waves propagating through at thick 1-D randomly layered stack embedded between two identical homogeneous halfspaces is studied. However, unlike in previous works, the restriction to the individual layers being isotropic is now being dropped, and intrinsic transverse isotropy is taken into consideration. Thus, two different kinds of anisotropy must be combined: [A] frequency-dependent anisotropy due to thin layering and [B] frequency-independent intrinsic anisotropy. To compare the significance of both effects for the transmissivity of seismic waves is a subject of the investigations. Analytical results are presented for different degrees of intrinsic anisotropy superimposing the effect of small-scale fluctuations of the medium parameters on wave propagation. The emphasis is thereby laid on the frequency-dependent shear-wave splitting.

INTRODUCTION

Multiple scattering at small-scale 1-D inhomogeneities causes an exponential attenuation of seismic waves, which strongly depends on the frequency and the angle of incidence and influences velocity dispersion and shearwave splitting. (Shapiro and Hubral, 1996) and (Shapiro et al., 1994) provided an analytical description of the qP, qSV, and SH-waves propagating in randomly multilayered media. Their so-called 'generalized O'Doherty-Anstey formulas' can be used for both deterministically and statistically specified stratifications. Their method not only holds for the entire frequency range, but also for oblique incidence. However, a restriction is the assumption that the fluctuations of

[1]**email:** uwerner@gpirs2.physik.uni-karlsruhe.de

the medium parameters are small compared to their average values. In the generalized O'Doherty-Anstey theory, the individual layers are assumed isotropic. The observed frequency-dependent anisotropic behaviour of the transmissivity in such media is exclusively caused by wavefield scattering due to thin multilayering. In this work, however, intrinsic anisotropy of the individual layers is also taken into account. We show that, unlike the anisotropy caused by multilayering, the intrinsic anisotropy does not depend on the frequency of the transmitted wave.

THE MODEL

In the case of transversely isotropic multilayering five elastic parameters are necessary to describe the elastic wavefield completely. In addition to the homogeneous P- and S-wave velocities α_0 and β_0 we use three 'anisotropy parameters' $\eta, \nu,$ and χ. Two of them, η and χ, are identical with the well-known Thomsen parameters ε and γ ((Thomsen, 1993)), respectively, and the third one, ν, has been designed for reasons of simplicity. If we consider a horizontally stratified medium with the z-axis being the symmetry axis (VTI-medium), η and ν describe the deviations of the qP/qSV-wave behaviour from the isotropic case as well as wave conversion effects, whereas χ is a measure for the difference between the vertical S-wave velocity β_0 and the horizontal velocity of the SH-wave. Our model consists of a thick 1-D inhomogeneous, elastic medium (e.g. a randomly multilayered stack) embedded between two identical homogeneous halfspaces characterizing a homogeneous, transversely isotropic reference medium with vertical symmetry axis. The actual parameters in the 1-D inhomogeneous medium may vary in z-direction in a random manner. However, the restriction applies that the parameter fluctuations and the anisotropy parameters must be small. All parameters are constant in the horizontal (x-y-) plane at any given z. The y-axis is normal to the incidence plane. From the uppermost halfspace a plane S-wave enters the inhomogeneous medium and is split into two shear-wave phases, the qSV-wave being polarized in the incidence plane, and the SH-wave perpendicular to it along the y-axis. Our aim is to evolve analytical formulas for the angle- and frequency-dependent shear-wave splitting in such media.

THEORY

In the following, the procedure that leads us to the analytical formulas for the time-harmonic transmissivities, from which the phase velocities and attenuation coefficients can be extracted, is briefly summarized. The strategy of solution is described in detail in (Shapiro and Hubral, 1996). Since there is no interaction between the qP/qSV-wavefield and the SH-wavefield, they can be studied separately. However, for both the same strategy can be applied: In the 1-D inhomogeneous medium the wavefield can be described by a vector $\vec{f}(z)exp\,[\,i\omega\,(px - t\,)]$ that satisfies the following first-order differential matrix equation:

$$\frac{\partial}{\partial z}\vec{f}(z) = \mathbf{Q}^{TI}(z)\vec{f}(z) \ . \qquad (1)$$

$\mathbf{Q}^{TI}(z)$ is a matrix that carries all information about the physical properties of the 1-D inhomogeneous medium. It contains the depth-dependent velocities $\alpha(z), \beta(z)$, density $\varrho(z)$, and anisotropy parameters $\eta(z), \nu(z), \chi(z)$. In the inhomogeneous region these parameters can be separated into averaged parts, e.g. $\alpha_0 = \langle \alpha_{true}(z) \rangle$ and appropriate fluctuating parts, e.g. $\varepsilon_\alpha(z)$ in the following manner: $\alpha_{true}(z) = \alpha_0(1 + \varepsilon_\alpha(z))$. This procedure can be applied for all medium parameters. The constant quantities, $\alpha_0, \beta_0, \eta_0, \nu_0, \chi_0$ and ϱ_0 , characterize a homogeneous reference medium obtained by averaging the depth-dependent medium parameters. Their fluctuating parts, $\varepsilon_\alpha(z), ...$ are a measure for the deviation of the 'real' medium from the averaged reference medium at any given depth z. Due to their definitions the mean value of these medium fluctuations is zero. As a consequence of the separation of the physical medium parameters into a homogeneous and a fluctuating part, eq. (1) can be rewritten in a new form:

$$\frac{\partial}{\partial z} \vec{f}(z) = \mathbf{Q}_0^{TI}\, \vec{f}(z) + \mathbf{Q}_\delta^{TI}(z)\, \vec{f}(z) \tag{2}$$

$\mathbf{Q}_0^{TI}$ is made up of the constant averaged quantities. It is, therefore, independent of z and represents the homogeneous transversely isotropic reference medium. The fluctuation matrices $\mathbf{Q}_\delta^{\mathbf{TI}}(z)$ consist of known combinations of terms with fluctuations of first-order and higher-order powers. By analogy with (Shapiro and Hubral, 1996), the time-harmonic transmissivity, which describes the response in the lowermost halfspace to a plane wave incident in the upper halfspace, can be expressed as follows:

$$T_{SV,SH} = \exp(i(\psi_{SV,SH}(p,\omega)L + \omega p x - \omega t) - \gamma_{SV,SH}(p,\omega)L)\,. \tag{3}$$

ω is the angular frequency, p the horizontal slowness, and L the thickness of the inhomogeneous part of the medium along the symmetry(z)-axis. $\psi_{SV,SH}$, the so-called 'vertical phase increment', denotes the real part of the vertical component of the wave vector, and $\gamma_{SV,SH}$ is the attenuation coefficient. The results for the vertical phase increments read:

$$\psi_{SV,SH} = \lambda_{SV,SH} + \omega\, A_{SV,SH} + \omega^2 \int_0^\infty d\xi\, B'_{SV,SH}(\xi, \lambda_{SV,SH})\,, \tag{4}$$

The quantities $\lambda_{SV,SH}$, which are proportional to ω, are the eigenvalues of the corresponding matrices $\mathbf{Q}_0^{TI}$; they represent the contribution of the intrinsic anisotropy of the background. $A_{SV,SH}$ contain variances and crossvariances of the medium fluctuations, describing the low-frequency Backus correction due to thin layering. And finally, $B'_{SV,SH}$ are made up of known combinations of the auto- and crosscorrelation functions of the medium fluctuations; the integral expressions characterize the frequency-dependent effect due to thin layering. The phase velocities $V_{SV,SH}(p,\omega)$ can be obtained from the vertical phase increments by:

$$V_{SV,SH}(\theta,\omega) = \frac{1}{\sqrt{p^2 + \frac{\psi^2_{SV,SH}(p,\omega)}{\omega^2}}}\,. \tag{5}$$

RESULTS AND CONCLUSIONS

Results are illustrated for the frequency-dependent shear-wave splitting at different angles of incidence for media with small and large intrinsic anisotropy. The shear-wave splitting is defined by: $S(\omega, p) = (V_{SV}(\omega, p) - V_{SH}(\omega, p))/\beta_0$. We consider a medium with an exponential correlation function. The parameters are given in the caption. The shear-wave splitting is illustrated for media without (Fig.1a) and with (Fig.1b) intrinsic anisotropy. A comparison of both cases shows that in the presence of intrinsic

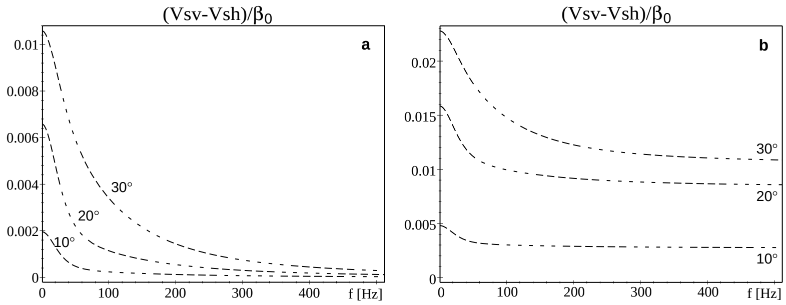

Figure 1: Frequency-dependent shear-wave splitting for $\theta = 10^o, 20^o, 30^o; \alpha_0 = 4000m/s, \beta_0 = 2325m/s, \varrho_0 = 2500kg/m^3, l = 5m, \sigma_{\alpha,\beta,\varrho} = 0.1$; left picture: no intrinsic anisotropy, right picture: strong intrinsic anisotropy ($\eta_0 = \nu_0 = \chi_0 = 0.1$)

anisotropy shear-wave splitting occurs even for high frequencies, whereas without intrinsic anisotropy it tends to zero in the high-frequency limit regardless of the incidence angle. The frequency-dependence is thereby not changed. From this we can conclude that the contribution of the intrinsic anisotropy is a shift of the shear-wave splitting to larger values by an amount that is determined by the magnitude of the intrinsic anisotropy. This shift depends only on the frequency if the intrinsic anisotropy is subject to fluctuations and becomes larger with increasing angle of incidence.

REFERENCES

Backus, G. E., 1962, Long-wave elastic anisotropy produced by horizontal layering: J. geophys. Res., **67**, 4427–4440.

O'Doherty, R. F., and Anstey, N. A., 1971, Reflections on amplitudes: Geophys. Prospect., **19**, 430–458.

Shapiro, S. A., and Hubral, P., 1996, Elastic waves in thinly layered sediments: the equivalent medium and generalized o'doherty-anstey formulas: Geophysics, **61**, 1282–1300.

Shapiro, S. A., Hubral, P., and Zien, H., 1994, Frequency-dependent anisotropy of scalar waves in a multilayered medium: J. seism. Expl., **3**, 37–52.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954–1966.