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What Higher Order Statistics can reveal on the Seismogram

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INTRODUCTION

Commonly, signal processing uses second order statistics, i.e., the autocorrelation or its Fourier Transform, the power spectrum. Additional information contained in higher order statistics (spectra) is lost. How can the variety of useful properties of higher order statistics be used in reflection seismics? Some suggestions were presented in (Weiss and Treitel, 1996) and shortly summarized below.

Firstly, higher order statistics provide phase information. Thus, the standard *minimum-phase* assumption can be dropped for deconvolution. Secondly, higher statistics are blind to white and colored Gaussian noise. Thirdly, autocorrelation methods can be generalized to higher order correlation methods. Therefore, deconvolution methods based on the concept of higher order cumulants have been developed (see, e.g., (Mendel, 1991),(Lazaer, 1993),(Velis and Ulrych, 1995) and (Cadzow, 1996)).

Investigations in the time domain proved to be of more value than those in the frequency domain which requires complicated phase unwrapping. Statistical measures in the time domain are cumulants, which are higher order correlation functions of a time series. Exploiting the third order cumulant of seismic traces proved to be not possible. It has been found that the fourth order cumulant clearly contain information that can be exploited. Restricting analysis to the fourth order cumulant of seismic traces is reasonable because of the rapidly growing variance of the estimate of the n-th order cumulant with increasing order n.

In (Weiss, 1996) it is studied in detail how well time series analysis with the fourth order cumulant can perform for seismic traces. For this analysis, the convolutional model (see (Robinson and Treitel, 1980)) and the stationarity assumption have been employed. The basic idea is to model the seismogram as an Auto-Regressive Moving-Average (ARMA) process ((Robinson and Treitel, 1980)). Fourth order statistics is capable of providing both ARMA filter order as well as the numerical values of the ARMA filter coefficients. Thus, supplementary information for deconvolution is provided. Figures

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1 - 4 demonstrate the performance of fourth order cumulant methods in multiple suppression and wavelet retrieval for a synthetic example. Investigations show that the use of fourth order cumulant methods gives promising results for synthetic data. The fourth order cumulants of reflection time series might well turn out useful for processing.

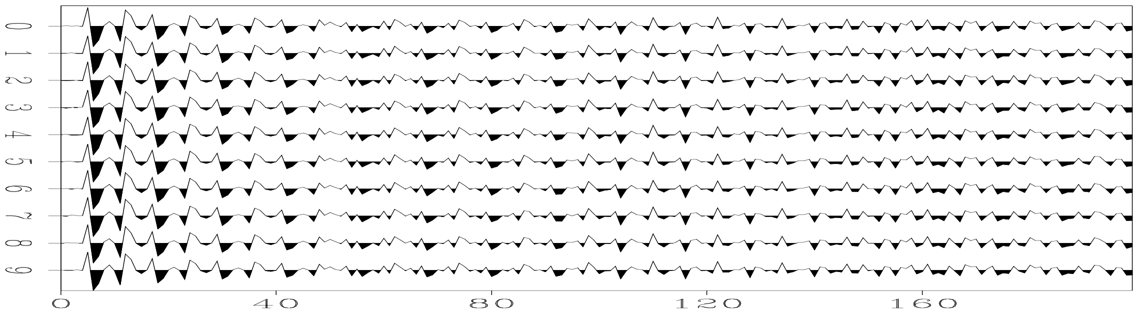
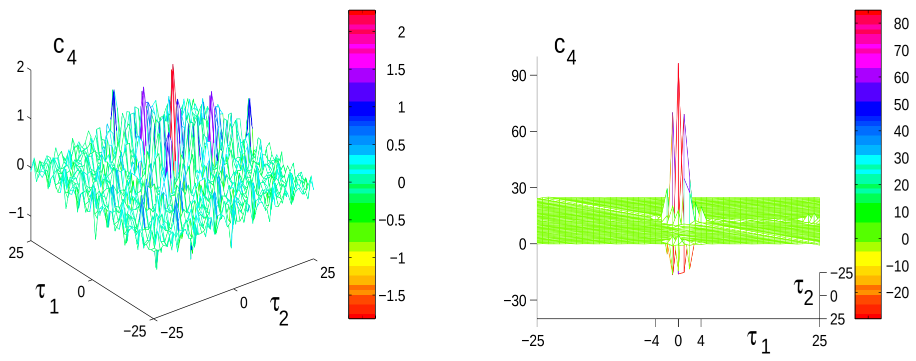


Figure 1: Synthetic data with strong multiples.



(a) (b)

Figure 2: a) A selected slice of its fourth order cumulant. b) A selected slice of its fourth order cumulant.

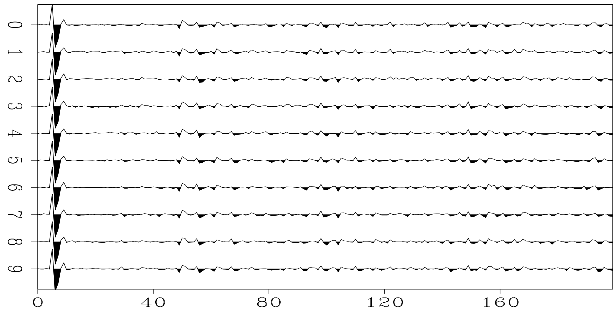
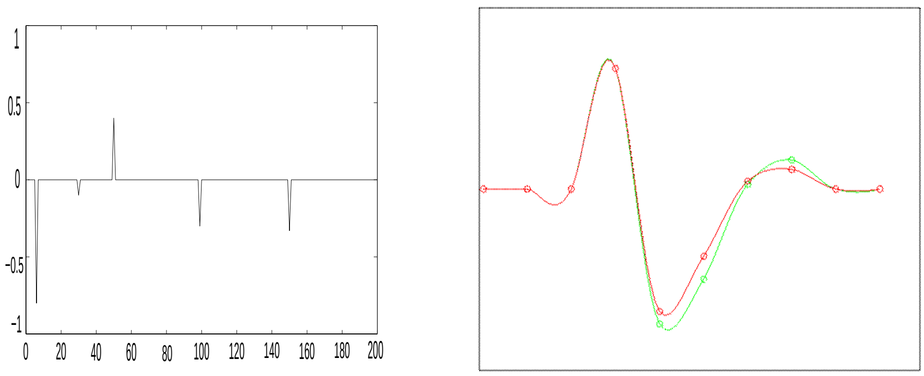


Figure 3: Residual time series (result of multiple suppression via fourth order cumulant methods).



(a) (b)

Figure 4: a) True primary series for comparison. b) True (green) and from fourth order cumulant estimated (red) wavelet.

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PUBLICATIONS

Detailed results were published by (Weiss and Treitel, 1996), (Weiss, 1996).