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# Seismic signatures of permeability

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## ABSTRACT

*In homogeneous poroelastic systems with physical parameters realistic for usual hydrocarbon reservoirs the permeability tensor practically does not influence propagating seismic waves in the low frequency range (0-1000Hz). Geological structures are, however, heterogeneous. In this paper using recent generalization of the O'Doherty-Anstey formalism to poroelastic media (Biot's model) we show that in heterogeneous systems the transport properties of rocks, as well as their symmetry, significantly affect seismic wavefields. Moreover, seismic signatures of the transport properties are essentially different in periodically heterogeneous media and in media with disorder.*

## INTRODUCTION

It is known that in homogeneous poroelastic systems the permeability tensor practically does not influence propagating seismic waves in the low frequency range (0-1000Hz; see, e.g., Gelinsky and Shapiro, 1996). In this paper we show that this situation changes in heterogeneous systems, like, e.g., layered or fractured sediments.

Our consideration is based on the recent studies of wave propagation in thinly layered poroelastic saturated or partially saturated sediments (Gelinsky and Shapiro, 1997, Gurevich and Lopatnikov, 1995 and Norris, 1993; these papers, in turn, are based on the Biot's theory).

In such structures two effects are of importance for the seismic attenuation and velocity dispersion in the frequency range mentioned above. The first one is the inter-layer (or local) flow, which is a small-spatial-scale fluid motion at interfaces in structures. This effect is due to generation of dissipative slow waves during scattering of seismic waves on the interfaces. It can also be understood as the process of diffusion (i.e., pressure relaxation) on the scale of heterogeneities. The second important effect is the (poro)elastic scattering of seismic waves, which is usual for heterogeneous elastic media.

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## ATTENUATION OF P-WAVES

Let us consider the normal P-wave vertically propagating in a randomly-layered saturated (or partially saturated) structure. For simplicity we assume that the statistics of heterogeneities is stationary and the standard deviations of physical parameters are small (of the order of 30 per cent). We also neglect the effects of density fluctuations, which are usually small. Finally, we restrict our consideration to the case of isotropic layers.

For such media we obtain the following formula for the reciprocal quality factor $Q^{-1}$:

$$Q^{-1} = \frac{\alpha^2 N \Delta_1}{4H} F_{flow}(x_{flow}) + \frac{N^2 \Delta_2}{4M^2} F_{scat}(x_{scat}), \qquad (1)$$

where $\alpha = 1 - K_d/K_g; M = (\phi/K_f + (\alpha - \phi)/K_g)^{-1}; N = MP_d/H; H = P_d + \alpha^2 M; P_d = K_d + 4/3\mu_d$; $K_{f,d,g}$ are bulk moduli of the fluid, dry frame and grain material respectively; $\mu_d$ is the shear modulus of the frame and $\phi$ is the porosity. In contrast to these statistically averaged poroelastic parameters the quantities $\Delta_{1,2}$ are measures of heterogeneity of the medium. They are two different linear combinations of the normalized variances and covariances of the poroelastic parameters. To describe the fluctuations of the poroelastic parameters we need also their correlation functions. For simplicity we assume that all correlation functions are equal to a single function $\Phi(\zeta)$,where $\zeta = \Delta z/a$ and $a$ is the characteristical size of the heterogeneities.

It is important to note that all parameters composing in $Q^{-1}$ the factors before the functions $F$ do not contain any dynamic information. Accordingly, they do not contain any information on the permeability. However, these parameters define the magnitude of the attenuation.

The dynamic dependence of the attenuation is contained in the quantities $F_{flow}(x)$ and $F_{scat}(x)$. Their arguments depend on frequency as follows: $x_{flow} = \sqrt{\omega a^2 r/2N}$; $x_{scat} = \omega a/c_0$, where $c_0$ is the low-frequency limit of the P-wave velocity, $\rho$ is the density of the saturated rock and $r = \eta/k$ is the hydraulic resistivity. Here $k$ denotes the permeability and $\eta$ stands for the dynamic viscosity of the fluid.

The second term in equation (1) describes the usual elastic scattering. It is important in the seismic-frequency range and was analyzed in the literature (see e.g., Shapiro and Hubral, 1996). In the following analysis we concentrate us mainly on the contribution of the inter-layer flow.

## FREQUENCY DEPENDENCE

The frequency dependence of the attenuation is fully defined by the functions $F_{flow}(x)$ and $F_{scat}(x)$, which are forms of Fourier transforms of the correlation function $\Phi(\zeta)$. Therefore, the frequency dependence is controlled by the correlation properties of the medium heterogeneities (i.e., by the disorder of rock structures).

Let us firstly consider a periodic structure (e.g., no disorder). Using symmetry properties of a periodic correlation function and considering the limit $x_{flow} \longrightarrow 0$ we obtain that $Q^{-1}$ is proportional to frequency. The high-frequency limit of the function $F_{flow}$ depends on individual properties of the function $\Phi$.

Let us now consider the general case of randomly heterogeneous media (i.e., media with disorder). In this case the correlation between any two points must decrease with increasing distance between them. Considering now the limit $x_{flow} \longrightarrow 0$ we immediately obtain: the $Q^{-1}$-contribution of the inter-layer flow is proportional to $\omega^{1/2}$ as well as such a contribution of the elastic scattering is proportional to $\omega$. This frequency dependence is an analogon of the Rayleigh-scattering frequency dependence and it is universal for all poroelastic media with disorder. Moreover, the low-frequency range attenuation coefficient is proportional to $k^{-1/2}$.

It is interesting to note that there is no universal behavior in the case of $x_{flow} \gg 1$, where the frequency dependence of the inter-layer-flow attenuation depends on statistics of the heterogeneities. Let us consider a possibly general case of fractal-like heterogeneities which are characterized by the von Karman correlation function. We obtain in the high-frequency limit ($x_{flow,scat} \longrightarrow \infty$)

$Q^{-1}_{flow} \propto \omega^{-\nu}, \quad Q^{-1}_{scat} \propto \omega^{-2\nu}$, where $\nu$ is related to the fractal dimension (e.g., for exponential media $\nu = 1/2$). The functions $F_{flow}(x)$ and $F_{scat}(x)$ are universal positive functions with magnitudes of the order $O(1)$, i.e., the magnitudes are independent on the values of the arguments $x = x_{flow}$ and $x = x_{scat}$. These functions reach their maxima at $x = O(1)$. Therefore, the permeability controls the location of the maximum of the inter-layer-flow part in the frequency range. This maximum is reached at frequencies of the order $O(2Nk/a^2\eta)$. In realistic situations, if $a$ is in the range of $10^{-2} - 10m$ the inter-layer-flow attenuation will be significant at least in a part of the seismic frequency range.

## HETEROGENEOUS PERMEABILITY

Until now we have considered situations with no fluctuations of the permeability. In reality, however, the permeability is a strongly fluctuating quantity. Therefore, the permeability must enter into equation (1) in a somehow averaged form. Here we immediately face the following problem: It is well known that in the static limit the permeability of 1-D heterogeneous media in the direction normal to the layering is given by the averaging $\langle 1/k \rangle$.

A simple numerical study shows, however, that the estimation $\langle k \rangle$ much better satisfies the attenuation observed (see Figure 1). This is explained by the following: The inter-layer flow is a local flow effect, where the fluid motion takes place not through the total system but around different heterogeneities in regions of the scale of the slow-wave wavelength. These regions provide different contributions to the dissipation of the elastic energy. The contributions are controlled by local values of the permeability. The propagating P-wave averages then all different contributions. Therefore, the following rule should be applied: $\beta_{ef} = \int_0^\infty \beta(k)f(k)dk$, where $\beta_{ef}$ is the resulting attenuation coefficient, $\beta(k)$ is the attenuation coefficient for the medium with a constant $k$ and $f(k)$ is the probability density of $k$. Thus, for normally distributed $k$ the estimation $\langle k \rangle$ is satisfactory.

Therefore, even in media with huge fluctuations of the permeability the results described here are applicable and the seismic signatures of the transport properties can be observed.

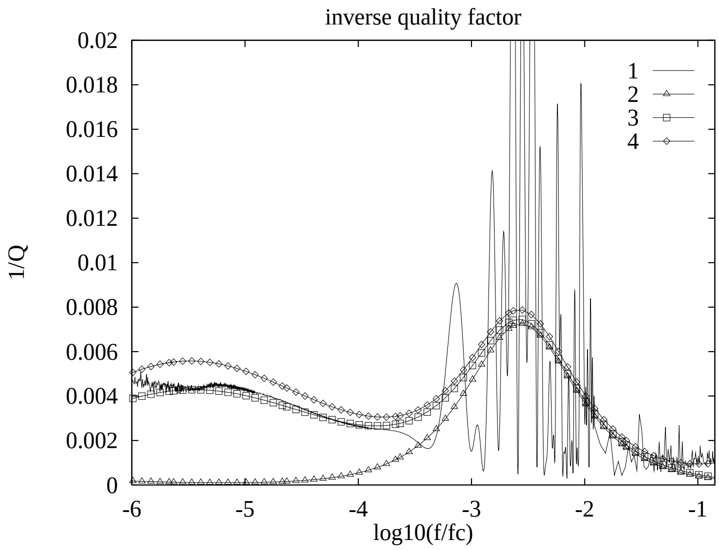


Figure 1: Inverse quality factor of the P-wave versus frequency in a randomly layered porous water-saturated (with a fluctuating gas concentration) medium. Line 1 shows results of numerical simulations (global matrix method for Biot equations; see Schmidt and Tango, 1986). Line 2 shows results of equation (1). The medium is characterized by the following average parameters: $c = 3600m/s, \rho = 2.4g/cm^3, \phi = 0.15$. The fluctuations are exponentially correlated with $a = 1m$ and $\sigma_{Kf} = 0.2, \sigma_P = 0.3, \sigma_\phi = 0.05$. The permeability was normally distributed with the standard deviation of $120$ per cent. Thus, the layer permeabilities range from $1\mu D$ up to $1D$. For computation of the theoretical curve the harmonic averaged value of the permeability $k = 7\mu D$ (i.e., hydraulic permeability) was used. Line 3 shows results of the suggested averaging rule. Normal averaging yields an average permeability $k = 300mD$ (this corresponds to a Biot critical frequency of $f_c = 81kHz$ ), which after substitution into equation (1) provides line 4.

## CONCLUSION

We have shown that in heterogeneous systems the transport properties of rocks significantly affect seismic wavefields. Moreover, seismic signatures of the transport properties are essentially different in periodically heterogeneous media and in media with disorder. The frequency dependence of the P-wave attenuation coefficient is sensitive to the permeability. This can be observed in the low-frequency (0-1000 Hz) range.

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