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Minimum Apertures and Fresnel zones in migration and demigration

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ABSTRACT

The size of the aperture has an important influence on the results of (Kirchhoff-type) migration and demigration. For true-amplitude imaging, it is crucial not to have apertures below a certain size. For both, the minimum migration and demigration apertures, theoretical expressions are established. Both minimum apertures depend on each other and, although a time-domain concept, are closely related to the frequency-dependent Fresnel zone on the searched-for subsurface reflector. This relationship sheds new light on the role of Fresnel zones in the seismic imaging of subsurface reflectors by showing that Fresnel zones are not only important in resolution studies but also for the correct determination of migration amplitudes. It further helps to better understand the intrinsic interconnection between pre-stack migration and demigration as inverse procedures of the same type. In contrast to the common opinion that it is always the greatest possible aperture that yields the best signal-to noise enhancement, it is in fact the selection of a minimum aperture that should be desired in order to (a) enhance the computational efficiency and reduce the cost of the summation, (b) improve the image quality by minimizing the noise on account of summing the smallest number of traces, and (c) to have a better control over boundary effects. This paper demonstrates these features rather than addressing the question of how to technically achieve them.

INTRODUCTION

Kirchhoff-type pre-stack migration based on summation and zero-order ray theory concepts is presently considered the most important 3-D migration scheme because it can be implemented on massively parallel computers at acceptable costs (Kao, 1992). Its inverse process called demigration recently also gained importance for the purpose of macro-velocity model determination, updating and validation (Fagin, 1994). Kirchhoff-type migration (demigration) can be mathematically described by a diffraction-stack (isochrone-stack) integral (Hubral et al., 1996; Tygel et al., 1996). Its efficiency and accuracy is, amongst other influences, strongly dependent on the migration (demigration) aperture.

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To carry out a diffraction-stack migration or isochrone-stack demigration in practice may become very computer-time consuming if large data volumes are to be imaged as is commonly the case in 3-D seismic surveys. Thus, the determination of a minimum aperture that leads to an optimum image with the least stacking effort is surely a desirable task. In this paper we address the important role which Fresnel zones have in this connection in the framework of the theory of pre-stack true-amplitude migration and demigration.

The problem is explained with the help of the following understandings. Suppose that an elementary seismic wave is excited at a point source at S, reflected at M_R, and recorded by a geophone at point G. It travels along the primary reflection ray SM_RG from S to G. The region of the reflector that influences this elementary reflection is its so-called Fresnel zone (for a certain frequency) on the reflector Σ at the reflection point M_R. The same concept applies to the primary reflected paraxial ray $\overline{S}\,\overline{M}_R\,\overline{G}$ from a point source at $\overline{S}$ in the vicinity of S to a geophone at $\overline{G}$ in the vicinity of G. The measurement configuration (e.g., common shot or common offset) is described, as quantified below, by a 2-D parameter vector $\boldsymbol{\xi}$ that uniquely defines one source location $\overline{S}(\boldsymbol{\xi})$ and one receiver location $\overline{G}(\boldsymbol{\xi})$ on the measurement surface. The parameter vector $\boldsymbol{\xi}$ serves as a coordinate vector in the seismic record section. All possible locations of source-receiver pairs $(\overline{S}(\boldsymbol{\xi}), \overline{G}(\boldsymbol{\xi}))$ define the range of possible values of $\boldsymbol{\xi}$ that is called the aperture A of the seismic experiment.

Diffraction-stack or Kirchhoff migration can be used to correctly image point M_R on the searched-for reflector Σ from primary seismic reflections along the reflection-traveltime surface Γ. To construct this image, one requires the diffraction-traveltime (or Huygens) surface for point M_R This is computed by treating point M_R as if it were a diffraction point. All rays $\overline{S}\,M_R\,\overline{G}$ define the Huygens surface of M_R. This surface is tangent to the reflection-time surface Γ in the seismic record section at the particular parameter vector $\boldsymbol{\xi} = \boldsymbol{\xi}^*$ that defines the source-receiver pair (S, G) for which the ray SM_RG is a specular reflection ray (Tygel et al., 1995a). The summation of all seismic traces along the Huygens surface within an a priori specified aperture A yields a pre-stack depth-migrated image of the reflector at M_R when placing the resulting sum into point M_R. If another point M in the (x, y, z)-space is chosen that does not fall into the depth-migrated strip of the reflector image, no significant summation value is to be obtained (Bleistein, 1987; Schleicher et al., 1993).

The size of the migration aperture A has an important influence on the migration amplitude that must not be underestimated. This has been investigated in various publications (Katz and Henyey, 1992; Hoxha, 1994; Krebs, 1994; Stolt and Benson, 1986). Most recently, Sun (1997) discussed the aperture boundary effects of Kirchhoff-type migration in great detail. To determine the optimum size of the migration aperture, one has to consider the trade-off between the necessity to guarantee sufficient constructive interference of the migrated signals (for which a certain aperture size is required) on the one hand and the desire to economize on the number of traces as well as to stack as little noise as possible on the other hand. In this paper, we show how a minimum aperture A_{min} can be determined so that a best possible migration amplitude at point M_R upon the subsurface reflector Σ is obtained. This is done by means of extending the concept of the *projected Fresnel zone*, which has been defined by Hubral et al. (1993) for a zero-offset configuration, to arbitrary measurement configurations. This concept plays a key role in pre-stack depth migration where the seismic reflection strip attached to the reflection-time surface Γ in the seismic record (i.e., in the $(\boldsymbol{\xi}, t)$-space) is imaged onto its related depth-migrated strip attached to the reflector Σ (Tygel et al., 1994).

The concept of a diffraction-stack migration can be easily extended to that of an isochrone-stack demigration. The set of all possible rays SM_IG with constant traveltime $t(S, M_I, G) = t$ defines the isochrone for the source receiver pair (S, G) and for that time t. This isochrone is tangent to the reflector at the reflection point M_R. A stack of the migrated image along the isochrone—the isochrone stack—describes then the inverse procedure to the diffraction stack, i.e., it describes the demigration (Hubral et al., 1996; Tygel et al., 1996). For the isochrone stack there exists a minimum demigration aperture E_{min} defined on the plane $z = 0$. As shown below using time-domain concepts, E_{min} is determined by the actual frequency-dependent Fresnel zone upon the reflector Σ for a characteristic frequency, as well as by the length of the source wavelet attached to Γ in the vicinity of the receiver G. In other words, the minimum apertures of migration and demigration, A_{min} and E_{min}, are related in the same way as the projected and the actual Fresnel zone of the reflector at M_R.

It is to be clearly stated that the problem of how the tangency points of Huygens and reflection time surfaces (or also of the isochrone and the reflector) can be determined in a practically feasible way remains yet unsolved. The principal purpose of this paper is to establish the basic relationships between minimum apertures and Fresnel zones so as to demonstrate that seismic migrated and demigrated images can be improved by correctly choosing the size of the aperture. Although the fundamental problem of determining the stationary point directly from the data in an economic way remains unsolved, it is nevertheless important to understand and quantify the minimum aperture concept as this is bound to play a significant role in future migration algorithms. By establishing this concept on a sound theoretical basis, we aim to pave the way for the complete solution to the problem of how to design an optimal migration (and demigration).

Note the fundamental role played by the Fresnel zone in pre-stack true-amplitude migration. As is well-known (Berkhout, 1984; Lindsey, 1989; Knapp, 1991), the Fresnel zone is a key concept in the study of lateral resolution of seismic migration. Moreover, we will see below that it is the projected Fresnel zone that determines the minimum migration aperture. In fact, Fresnel zones do not only play a role in the migration of seismic reflections, but also in the demigration process. The demigration aperture is directly related to the Fresnel zone upon the reflector Σ which is to be imaged. In these considerations, the concept of the Fresnel zone, which was originally introduced in the frequency domain (see, e.g. Sommerfeld, 1964), becomes a time-domain meaning where the role of the period of a monofrequency wave is now played by the length of a transient signal. This time-domain meaning is the same as introduced in Knapp (1991) as a *Fresnel zone for broadband data* but differs from the recent one of Brühl et al. (1996).

CONCLUSION

Like any other migration (and demigration) methods, Kirchhoff-type migration depends on the size of the chosen migration aperture. However, whereas in other methods like f-k migration the migration aperture should be as large as possible, we have shown in this paper that this is not the case in Kirchhoff-type migration. If the measurement aperture is larger than necessary, it can be restricted to the actual region of tangency, where the diffraction and reflection traveltime surface “strips” touch, because it is from this region where all necessary information is actually gathered in a diffraction-stack migration. The restriction to that region not only economizes on the summation procedure, but it also enhances the S/N ratio when summing up noisy traces. Therefore, in the case of true-amplitude migration, where suitable weights are applied in the summation, this will result in a more reliable reconstruction of the interface reflection coefficients in dependence on the reflection angle. In this work, we have shown that the accurate control over the migration aperture becomes a must when stacking noisy traces.

Basic to this was the understanding of how the minimum migration and demigration apertures actually depend on the properties of the reflector an the overburden. The time-domain concepts of minimum apertures can be related to the frequency-dependent Fresnel zone. The role played by the period of a monofrequency wave in the definition of the Fresnel zone is taken by the length of the wavelet. The size of the minimum migration aperture is determined by the very same matrix that also defines the projected Fresnel zone in the seismic record. In the same way, the minimum demigration aperture is given by the Fresnel-zone matrix itself. We extended the concept of a projected Fresnel zone to arbitrary offset rays. Furthermore, we also presented a 3-D inversion method to compute the projected Fresnel zone of a primary reflection for any arbitrary seismic measurement configuration. The information that is needed for this computation is obtained from two traveltime surfaces. The first one is the reflection traveltime surface that is picked from the data. The second one is the diffraction traveltime surface that is constructed by means of a macro-velocity model. However, no knowledge about the reflector is needed. Projected Fresnel zones can consequently be computed with almost no extra effort, when a diffraction stack migration is to be performed. Because of the direct relationship between the projected Fresnel zone and the minimum diffraction-stack migration aperture, the latter can be computed during a diffraction-stack migration. As indicated earlier, there is still the fundamental and unsolved problem to find a technically feasible method for the determination of the tangency point where the minimum aperture is to be centered. However, even without a direct application in the stack, the minimum aperture can serve as a measure of reliability of migration (or demigration) amplitudes. Amplitudes obtained with apertures smaller than the derived minimum ones have no meaning.

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PUBLICATIONS

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