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Reflectivity modeling: a tool for testing processing algorithms

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ABSTRACT

The reflectivity modeling technique provides a fast and flexible tool for seismic modeling. It allows investigation of different aspects of specific wavefields. With the possibility of selectively switching on various wave types wave propagation aspects (conversions, multiples) can be investigated with ease. The method enables the calculation of synthetic sections for different source and receiver configurations, e.g VSP. Furthermore, the use of receiver arrays over a certain $x - z$ area is possible, such that time slices or arbitrary cuts through the wave field can be obtained. Computation of $\tau - p$ sections is a by-product of the modeling technique. Its flexibility allows to easily test assumptions on which processing algorithms are based. In contrast to other modeling techniques, such as Finite Difference modeling, the reflectivity method has the advantage of being fast, although restricted to laterally homogeneous media.

INTRODUCTION

Seismic processing algorithms are usually based on several basic assumptions about properties of the data, such as statistics, wave types, medium types, reflection or scattering geometries. If the assumptions are fulfilled the algorithms are supposed to work best, if assumptions are violated, then seismic processing algorithms can break down.

In order to asses viability and efficiency of seismic processing algorithms, they are always tested on synthetic data in an experiment in which all variables are totally controlled. Generating synthetic data which violate or fulfill a processing algorithms assumptions, is then a necessary task in order to quantify performance.

The reflectivity method is an efficient method for calculating synthetic seismograms for a laterally homogeneous earth. It is an excellent analysis tool if we want to investigate different wave types, such as converted waves or multiples. Such a selective investigation allows for kinematic as well as dynamic properties. Firstly, those wave types can be generated and visualized to gain insight in wave propagation mechanisms and secondly comprehensive test data sets for different kinds of subsurface models can be computed

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efficiently. Those test data sets might facilitate the study of multiple suppression methods or deconvolution operations, to mention only a few.

REFLECTIVITY TECHNIQUE

The reflectivity technique was designed for the calculation of synthetic seismograms from a point source in horizontally stratified isotropic media. The technique was extended to include weakly anisotropic layers by Nolte (1988) based on the work of Booth and Crampin (1983).
The wave field radiating from a point source is decomposed into plane waves characterized by the horizontal slowness p. The expression for the far–field component displacement spectra $u(\omega)$ at $(x, 0, z)$ can be written in terms of a Sommerfeld integral (Booth and Crampin, 1983):

$$u(\omega) \;=\; F(\omega)\sum_{j=1}^{3}\sum_{l=1}^{6}\int_{0}^{\infty} J_0(\omega x p) f^l a_j^l \, e^{-i\omega p z s_3^l} e^{-i\omega x p} dp \tag{1}$$

where J_0 is the Bessel function of first kind and zero order and $F(\omega)$ the source spectrum. The terms a_j^l and s_3^l refer to the components of the polarization vector and the vertical slowness of the different wave types l, respectively. The excitation factors f^l define the amplitudes of the different plane waves including the source effects and effects of all reflections, transmissions and mode conversions in the layer stack associated with the path dependent phase shift. They depend on the frequency, horizontal slowness and the reflection and transmission coefficients at the layer boundaries.

For the calculation of the plane wave response the iterative technique of Kennett and Kerry (1979) is used. This formulation includes variable source receiver combination and all free surface effects. For the recursion algorithm the excitation factors f^l are partitioned into Up– and Downgoing waves: l=1,2,3 denotes down– and l=4,5,6 upgoing wave types. The propagation of the plane waves through a layer stack is described by reflection and transmission matrices R_D^m, R_U^m, T_D^m and T_U^m and phase matrices D_D^m and D_U^m for each layer m. The reflection and transmission matrices

$$R_{U/D}^m \;=\; \begin{pmatrix} r_{pp} & r_{s_1p} & r_{s_2p} \\ r_{ps_1} & r_{s_1s_1} & r_{s_1s_2} \\ r_{ps_2} & r_{s_2s_1} & r_{s_2s_2} \end{pmatrix} \tag{2}$$

$$T_{U/D}^m \;=\; \begin{pmatrix} t_{pp} & t_{s_1p} & t_{s_2p} \\ t_{ps_1} & t_{s_1s_1} & t_{s_1s_2} \\ t_{ps_2} & t_{s_2s_1} & t_{s_2s_2} \end{pmatrix} \tag{3}$$

define the reflection– and transmission–coefficients (p, s_1 and s_2 refer to the different wave types) at the layer boundaries whereas the phase matrices

$$D_D^m \;=\; \begin{pmatrix} e^{i\omega s_3^1 d_m} & 0 & 0 \\ 0 & e^{i\omega s_3^2 d_m} & 0 \\ 0 & 0 & e^{i\omega s_3^3 d_m} \end{pmatrix}$$

$$D_U^m = \begin{pmatrix} e^{i\omega s_3^4 d_m} & 0 & 0 \\ 0 & e^{i\omega s_3^5 d_m} & 0 \\ 0 & 0 & e^{i\omega s_3^6 d_m} \end{pmatrix}$$

propagate the waves through the layer m with thickness d_m. Suppression of wave type conversions is achieved by setting the non–diagonal elements of the reflection and transmission matrices to zero.

In order to eliminate free surface multiples the reflection coefficients for reflections at the free surface are set to zero.
All effects of internal multiples and associated mode conversions of a layer m are described by a term

$$(I - A_m)^{-1} = I + \sum_{j=1}^{\infty} A_m^j \qquad (4)$$

where I is the 3×3 unity matrix and A_m a product of reflection matrices (Kennett, 1974). The expansion in eq.(4) corresponds to the set of physical rays in this layer. By truncating the series expansion the set of ray can be restricted to direct waves only (no internal multiples).

Subsequently we developed it further to acquire VSPs, and a allow for receiver arrays, thus allow a complete sets of wave fields to be generated.

$\tau - P$ TRANSFORM

An approach to separate overlapping seismic phases is the $\tau - p$ transform. Whereas in a typical seismic record a trace represents a particular offset each trace in a $\tau - p$ sections represents different rays characterized by the horizontal slowness p. The $\tau - p$ transform provides an alternative domain for separation of interfering reflections, refractions and conversions as well as a separation of pre– and postcritical arrivals (Tatham, 1989 and Diebold, 1989). Reflection hyperbolae are mapped to ellipses and refractions to points. This transform corresponds to a plane wave decomposition of the wave field. The reflectivity method is an excellent tool to calculate synthetic $\tau - p$ sections. The reflectivity function calculated by the recursion algorithm for each horizontal slowness is the required zero offset reflection time τ. Thus $\tau - p$ sections can be calculated very quickly with the same reflectivity code.

APPLICATION

In order to demonstrate the flexibility of this method we used synthetic models which are based on realistic geological settings. The model consists of a few layers with the first being the water layer. Density and p– and s–wave velocities increase with depth. The reflection coefficient at the sea bottom is about 0.3.

The following synthetic sections are computed using a ricker wavelet with a dominant frequency of 50 Hz and no direct arrivals. Multiples and mode conversions are switched on and off.

Figure 1 shows the full wavefield containing all conversions and multiples. The effect of periodic free surface multiples is obvious in the events with a approximate temporal periodicity of 260 $msec$. In contrast figure 2 a) depicts only the primary reflections. We see seven reflectors: the lowermost reflections are very weak where as the water bottom provides the strongest amplitudes. The sequence of the Figures 2 b) – 4 a) demonstrate the effects of wave type conversions and multiples. Figure 2 b) shows the primary reflections and associated conversions. The converted phases become very clear at offsets greater than 600 m. If we look at figure 3 a), we notice that there is almost no effect of internal multiples. From figure 3 b) it becomes clear that the dominant multiple phases are caused by the free surface multiples.

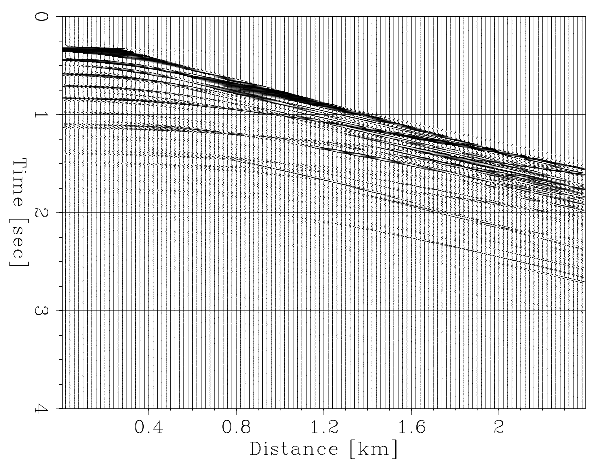

Figure 1: *Full wavefield containing all multiples and conversions.*

However, it is often difficult to distinguish between the different phases. The $\tau - p$ plots visualize a separation of primaries, mode conversions and multiples (Fig. 4–5). Each p value corresponds to a known angle of incidence and represents a single plane wave. If we consider the primary reflections in the $\tau - p$ domain (Fig. 4 a)), we see a clear separation of all seven reflections. The dominating effect of the water bottom becomes very clear. Figure 4 a) corresponds to Figure 2 a) in the $x - t$ domain. Figure 5 a) and b) demonstrate very well the separation and periodicity of the multiples. Note that the multiples are generated near the critical points.

For understanding wave propagation effects, it is often useful to view time slices of the wave field, in order to trace different phases of the wave field through the medium. By using a receiver array in a certain offset and depth range, time slices and the corresponding seismograms in $x - t$ and $z - t$ can be plotted. This is depicted in Figure 6. A

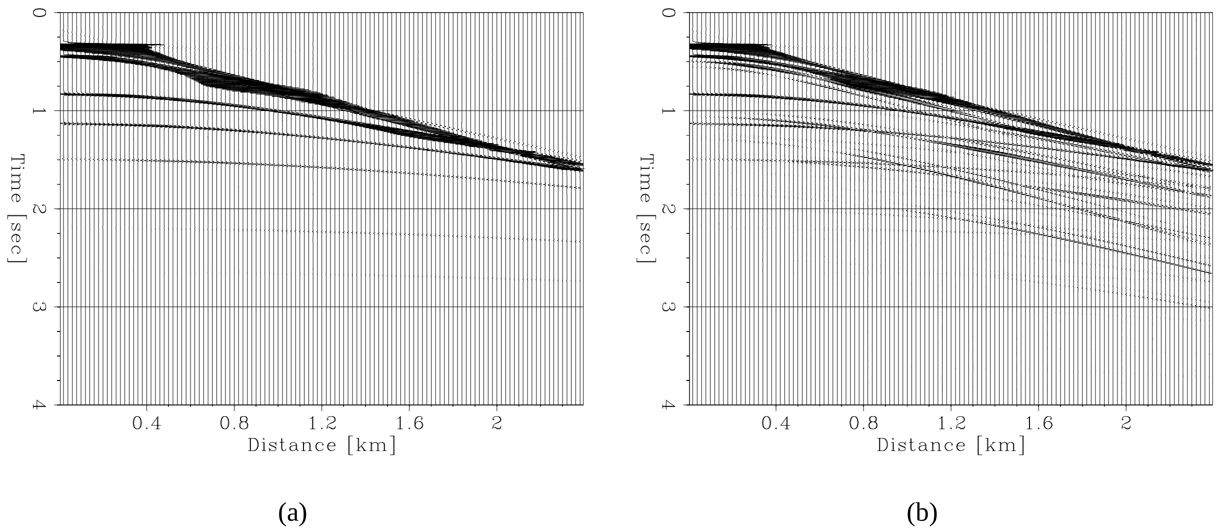

Figure 2: *a) By switching of multiples and conversions the primary reflections become clear. b) Mode conversions interfere especially in overcritical distance with the primaries.*

simple two layer model with the layer boundary at 200 m was used for this computation. The receiver spacing in x and z (e.g. surface and depth) is 10 m with a maximum offset of 500 m in each direction. On the front face a time slice at $t = 0.4\ s$ constructed from the traveltimes of the receiver array is shown. The top face displays the seismogram in $x - t$ with the receivers at the surface, where as the left face shows the $z - t$ section with the receivers at x= 10 m.

DISCUSSION AND CONCLUSIONS

Reflectivity modeling provides a fast and flexible tool for modeling seismic data in laterally homogeneous media. It allows selective investigation of various aspects of wave propagation. The method enables the calculation of synthetic sections for different source and receiver configurations, surface seismics, VSP, or the complete subsurface wave field. Computation of $\tau - p$ sections is a by-product of the modeling and allows a to easily test assumptions on which processing algorithms are based, such as deconvolution operations or multiple suppression schemes.

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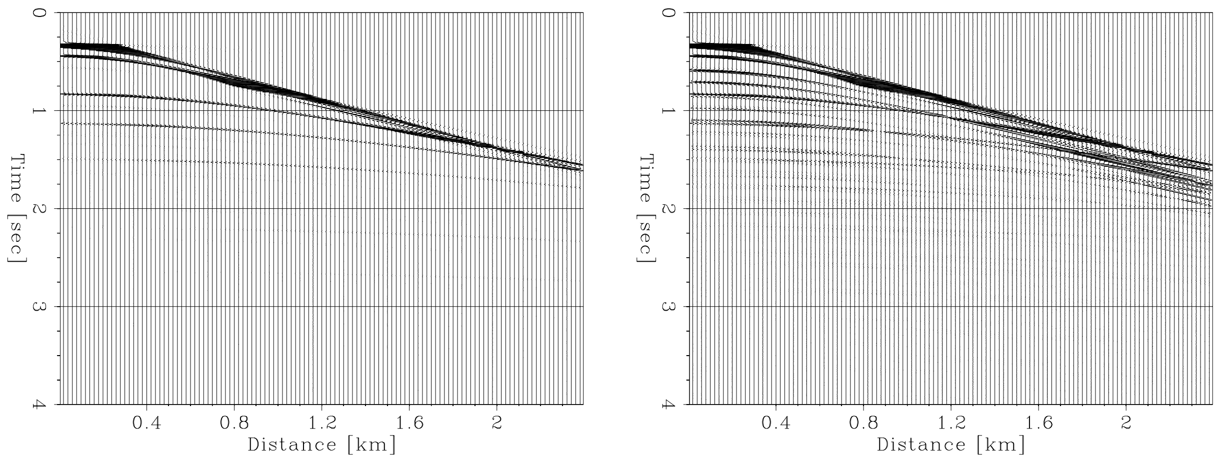

(a) (b)

Figure 3: *a) Primaries and internal multiples: there is almost no difference to Fig. 2a). b) Switching on the free surface multiples changes the wavefield drastically. The periodic free surface multiples interfere with the primaries.*

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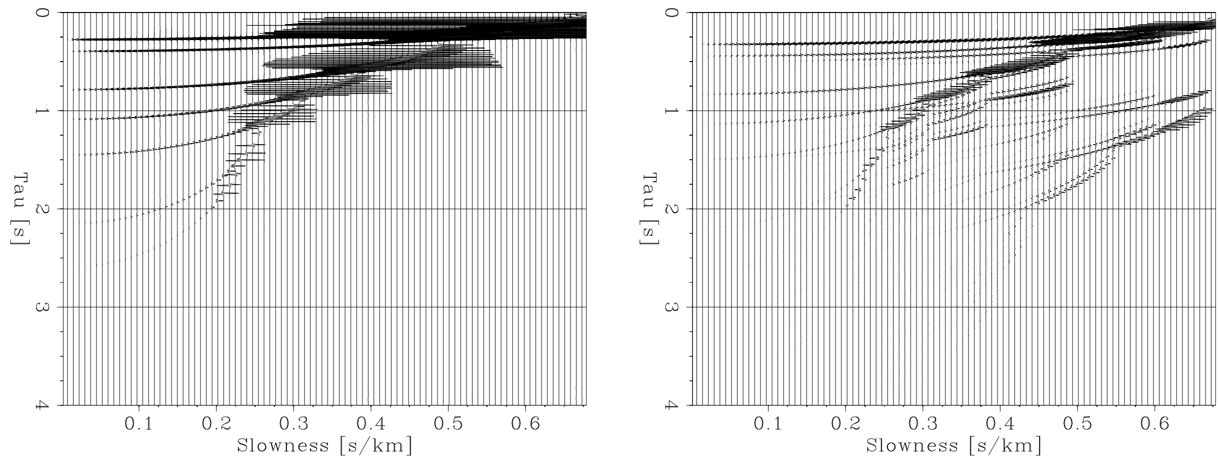
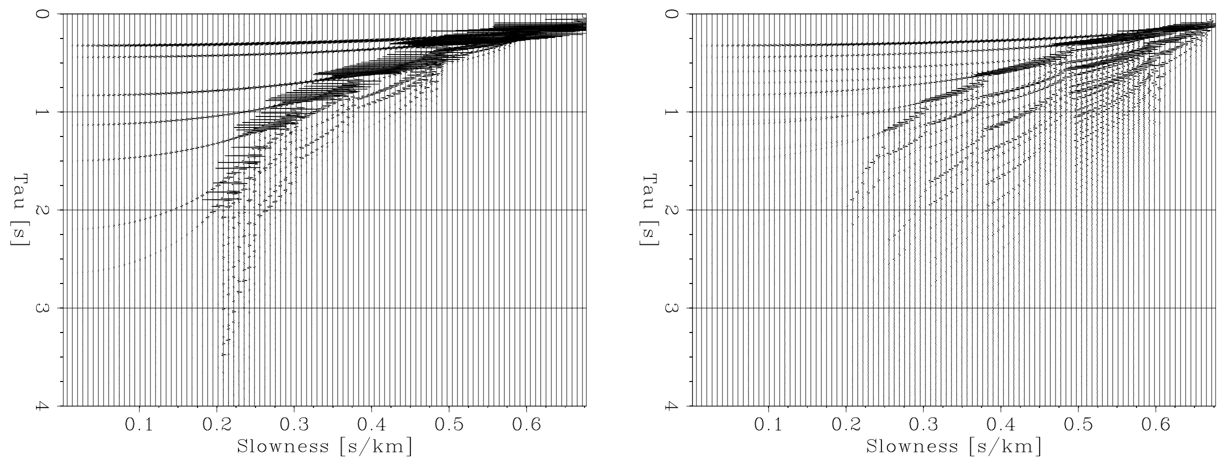


(a) (b)

Figure 4: a) $\tau - p$ *section corresponding to Fig.2 a). Each trace represents a single ray parameter* p*. The primary reflections are clearly separated.* b) $\tau - p$: *primaries and mode conversions.*



(a) (b)

Figure 5: a) $\tau - p$: *primaries and internal multiples, the multiples are generated at near the critical points.* b) $\tau - p$: *primaries and all multiples, the dominating effect of the free surface multiples can be seen clearly.*

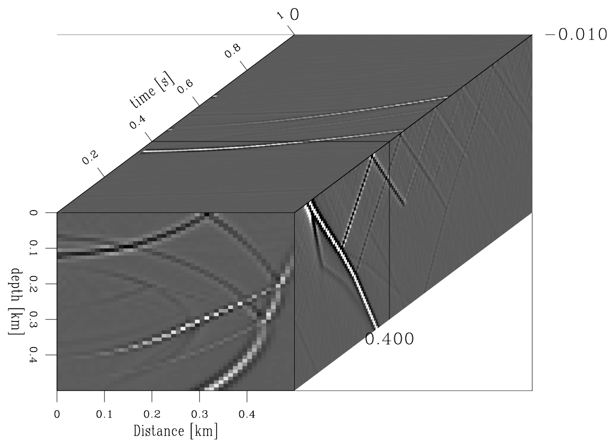


Figure 6: *Time slice at t= 0.4 s and corresponding sections in* $x - t$ *(top face) and* $z - t$ *(right face) with receivers at surface and at x= 10 m, respectively.*