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Computation of frequency-dependent traveltimes

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ABSTRACT

The eikonal equation for modeling ray propagation in inhomogeneous media is frequency independent. Therefore, it cannot correctly model the propagation of seismic waves when rapid variations in velocity cause frequency dispersion of the wavefield. Advantages and disadvantages of three methods which are extensions of frequency independent ray tracing to a frequency dependent process are presented.

INTRODUCTION

A commonly used method for computing traveltimes is given by the propagation of rays. The asymptotic ray tracing (ART) has advantages over full waveform methods because of its efficiency and the simple interpretation of results. There are many applications, e.g., it is used for modeling, in seismic tomography and in Kirchhoff migration.

However, application of ray tracing is limited to seismic signals with wavelengths small compared to characteristic lengths of the medium and of the wavefield itself. Reason is that the eikonal equation is a high frequency approximation and, therefore, frequency independent. This may lead to poor imaging in rather complex models when ray tracing is used for pre-stack Kirchhoff migration, since dispersion, scattering and other effects are neglected. In this case, the process of wave propagation must be described in dependence on frequency as it is done by numerically solving the wave equation or the elastodynamic equation of motion. Since this is not practical for many applications we present three different methods which extend kinematic ray tracing to a frequency dependent process. The methods by (Zhu and Chun, 1994) and (Biondi, 1992) involve solving the frequency dependent eikonal equation while the method by (Lomax, 1994) simulates the propagation for lower frequencies by a smoothing of the model.

In this paper these methods are called:

- perturbation scheme by Zhu
- extrapolation along the frequency axis by Biondi

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- smoothing of velocities by Lomax

PERTURBATION SCHEME BY ZHU

The acoustic wave equation for a general inhomogeneous medium with a constant density

$$\nabla^2\phi(\vec{x},t) - \frac{1}{v(\vec{x})^2}\frac{\partial^2\phi(\vec{x},t)}{\partial t^2} = 0 \tag{1}$$

is Fourier-transformed into the Helmholtz equation. $\phi(\vec{x},t)$ is wavefield and $v(\vec{x},t)$ the velocity of the medium at a point $\vec{x}$ and the time t. A harmonic ansatz and ordering terms with respect to real and imaginary part leads to the frequency dependent eikonal equation and the transport equation

$$(\nabla\tau)^2 = \frac{1}{v^2} + \frac{1}{\omega^2}\left(\frac{\nabla^2 A}{A}\right) \quad , \qquad \nabla^2\tau + \frac{2\nabla A\nabla\tau}{A} = 0 \quad , \tag{2}$$

with amplitude A, traveltime τ and frequency ω. By expansion of τ and the Jacobian in A to second order in ω (Zhu and Chun, 1994) derives

$$\frac{1}{v_f^2} = \frac{1}{v^2} + \omega^{-2}\left[-\frac{1}{4}\left(\frac{\nabla v}{v}\right)^2 + \frac{1}{2}\frac{\nabla^2 v}{v}\right] \quad , \tag{3}$$

where the frequency dependent velocity v_f is composed of the medium velocity v and the bracketed term which is considered small. Thus, Zhu develops a perturbation scheme which models well the effects in a constant velocity gradient medium (Zhu and Chun, 1994). Here we tested the method on the model of a circular shaped positive Gaussian velocity anomaly situated in the center of a constant velocity field. Figure 1 shows that velocity increases considerably with decreasing frequency. However, due to a smoothing effect for lower frequencies one would expect quite the opposite behavior. Thus, the method can' t be used for our purposes.

EXTRAPOLATION ALONG THE FREQUENCY AXIS BY BIONDI

(Biondi, 1992) extrapolates the velocity function along the frequency axis, starting from infinite frequency where the velocity is equal to the medium velocity. He avoids solving the coupled problem of equation (2) by considering the Jacobian of amplitudes A as constant. The theory of implicit functions leads to an equation for the derivative of velocity with respect to $\phi = 1/\omega$:

$$\frac{\partial v}{\partial\phi} = \frac{\phi[\frac{1}{4}(\nabla v)^2 v - \frac{1}{2}v^2\nabla^2 v]}{1 + \phi^2[\frac{1}{4}(\nabla v)^2 - \frac{1}{4}v\nabla^2 v]} \quad . \tag{4}$$

However, it turned out in our investigation that for small steps in frequency this method equals the method by (Zhu and Chun, 1994). Therefore, for the model of the Gaussian anomaly described above we also get non-physical results.

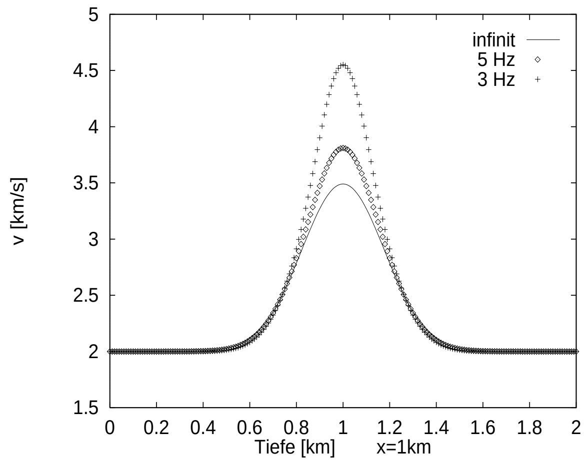

Figure 1: A vertical slice of the velocity function through the center of a Gaussian anomaly for three different frequencies computed by the perturbation scheme of Zhu.

SMOOTHING OF VELOCITIES BY LOMAX

This method is based on the computation of wave propagation using Huygens' principle and a frequency dependent velocity smoothing perpendicular to the wavefront. In a 2-D geometry the wavelength-averaged velocity $\bar{v}$ at point $\vec{x}_\nu$ and for period $T = 2\pi/f$ is given by

$$\bar{v}(\vec{x}_\nu, T) = \frac{\int\limits_{-\infty}^{\infty} w(\theta) v[\vec{x}(\theta, T)] d\theta}{\int\limits_{-\infty}^{\infty} w(\theta)} \quad , \tag{5}$$

where θ is distance from $\vec{x}_\nu$ along the wavefront expressed in wavelengths, $v(\vec{x})$ is the local medium velocity, and $w(\theta)$ is a Gaussian weighting function. To test the effect of the smoothing function the ray tracing results of this method are compared with a snapshot obtained by wave equation modeling using finite differences. A Ricker wavelet with a center frequency of 2 Hz is used for the latter method. Figure 1 shows that a wavefront computed by a 2 Hz velocity smoothing (solid line) fits better to the snapshot than the high frequency wavefront (dashed line).

CONCLUSION

Result of our investigation is that the method by (Lomax, 1994) can potentially be used for the computation of frequency-dependent traveltimes. Future work should be devoted to the determination of optimal parameters (width of smoothing, weights) to apply this method to Kirchhoff migration in complex media.

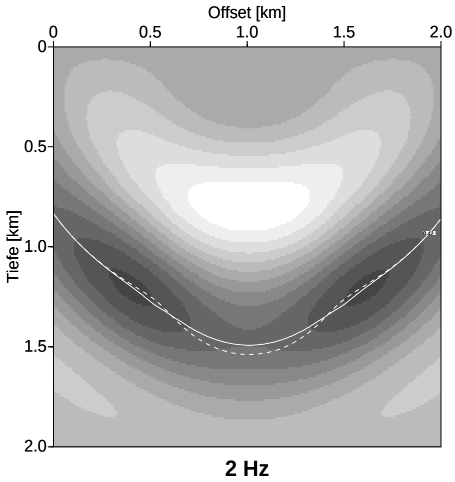

Figure 2: Snapshot at 0.42 s (gray scale plot); wavefront for a 2 Hz velocity smoothing (solid line) and high frequency wavefront (dashed line).

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PUBLICATIONS

Detailed results were published by (Koslowski, 1997) (in german).