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Upscaling in elastic random media

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ABSTRACT

In the case of complex heterogeneous media, the application of smoothing methods is often required for ray tracing and imaging. Often used smoothing methods are the averaging of the slowness or the squared slowness. However, these methods are not able to take into account the difference between scattering caused by fluctuations of the density and by the Lamé parameters. So in the case of elastic media we have to derive a new smoothing method that considers all medium parameters of the elastic medium in the right way. In this paper we suggest to treat the geophysical problem of the optimum smoothing (i.e., upscaling) of heterogeneous elastic media as a problem of homogenization theory. It is possible to reduce the problem of homogenization to the problem of calculation of the coherent wavefield (meanfield) in the low frequency limit. So after calculating analytical expressions for the coherent wavefield in elastic isotropic random media, we obtain a smoothing algorithm.

INTRODUCTION

In seismic processing it is often very useful to smooth acoustic or elastic media for the later application of imaging or ray tracing methods (Grubb and Walden, 1995). Since ray tracing methods can be successfully applied only on media with spatial fluctuations larger than the wavelength (Cerveny et al., 1977), smoothing is necessary when the medium contains small scale inhomogeneities. However, the main problem in smoothing elastic media is to find a method to average the elastic parameters in a way that the difference between the wavefield in the smoothed and original medium is minimized. Since fluctuations of the Lamé parameters and the density show different angular scattering behavior, averaging methods using only the velocities can't be able to average the medium in a way that both stiffness and density fluctuations are properly taken into account. Here we suggest to treat the geophysical problem of the optimum smoothing (i.e., upscaling) of heterogeneous elastic media as a problem of the homogenization theory. The mathematical background of this theory can be found in the classical books of (Bensoussan et al., 1978) and (Sanchez-Palencia, 1980). It can be shown that instead of solving the

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homogenization problem we can look for the coherent wavefield in the low-frequency range. Assuming additionally that the heterogeneity is weak, a general analytical recipe for the upscaling in the case of arbitrary isotropic structures can be obtained. In this paper we derive the coherent wavefield in elastic random media by means of the elastic Bourret approximation. Using the low frequency limit of this theory (see Appendix A), we present a frequency dependent way of smoothing, where the smoothing area is small compared with the wavelength, so that the result of the low frequency limit of the Bourret approximation can be applied.

THE METHOD

With all the considerations in the previous sections, we arrive at the following smoothing method for a point in an isotropic elastic random medium: at first we determine the averaged medium parameters and their (cross-)correlation functions in a circular area around this point. Using the grid points in this area we determine the mean values, the variances and the covariances of the elastic parameters. Equations (A-1) - (A-4) are then used to calculate the effective wavenumbers for this point. Using the averaged density, we can replace the real medium parameters by the upscaled medium parameters.

NUMERICAL RESULTS

To demonstrate our smoothing method and to determine the maximum smoothing volume around a point, we made several numerical tests using a finite difference program. For reasons of computing time, modeling was done in two dimensions and so the 2D smoothing algorithm was used.
For all numerical simulations, we used the same background medium, with a P-wave velocity of $4000m/s$, $2300m/s$ S-wave velocity and $2.5g/cm^3$ density. Upon this background we added fluctuations of several elastic parameters. Significant frequencies of the incident wavefield were in the range from 35 Hz up to 150 Hz with a maximum in the amplitude spectrum at the frequency of 75 Hz.

Example: Fluctuations of λ and μ

The following medium shows fluctuations of the Lamé parameters λ and μ, but no density fluctuations. In Figure 1, we see the medium parameters in a depth of 200 m.

Applying the 2D smoothing algorithm on the original medium, we obtained the smoothed medium shown in Figure 2. Note the change in axis description in comparison with Figure 1.

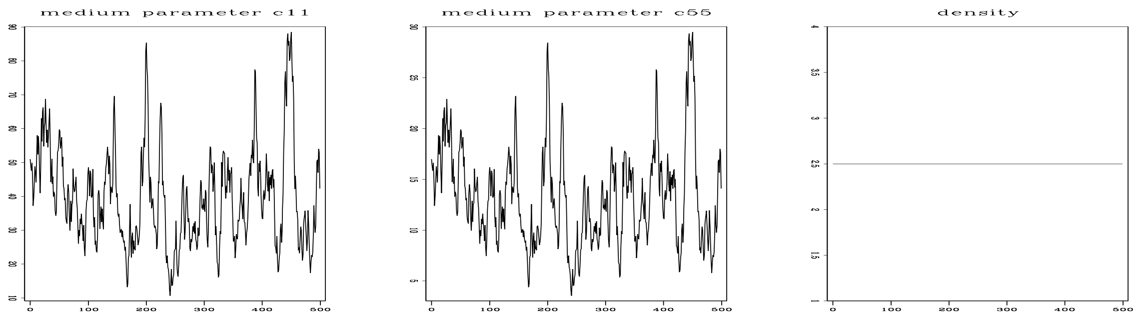

Figure 1: Parameters of Medium 1 in a depth of 200 m. The standard deviation of the stiffness tensor elements is 20 %, c11 and c55 ($= \mu$) are 100 % correlated.

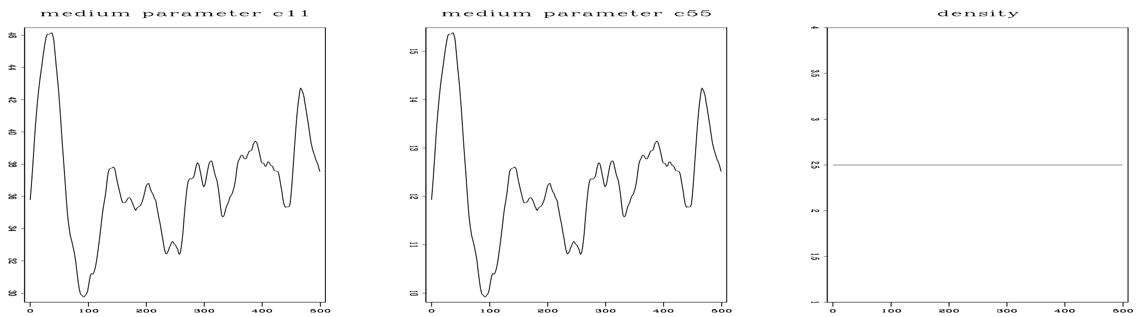

Figure 2: Parameters of Medium 1 after smoothing using the algorithm in equations (A-3) and (A-4). Note the significantly reduced fluctuations.

To compare the seismic records, we stacked them along the horizontal axis, resulting in a single seismogram. In the next Figure we see the stacked seismic sections of the original and the smoothed media.

We see a very good agreement in traveltime between the unsmoothed medium and the medium smoothed by using equations (A-3) and (A-4).
It should be noted that in the case of this special medium, the smoothing method using equations (A-3) and (A-4) results in a medium slightly faster than the medium we obtain by averaging the slowness.

CONCLUSION

The analytical description of wave propagation in random media, found by the Bourret approximation, gives us a quick and physically justified smoothing algorithm for elastic isotropic media in the low frequency limit. In contrast to methods that average the slowness or the squared slowness, our method is sensitive to the kind of medium fluctuation (ρ, λ or μ) and, since the smoothing radius depends on the wavelength, our method is also frequency dependent. Using synthetic data obtained by finite difference modeling, our smoothing method showed very good results if the smoothing radius was small enough and the medium fluctuations were isotropic. As a general rule, we expect our method to

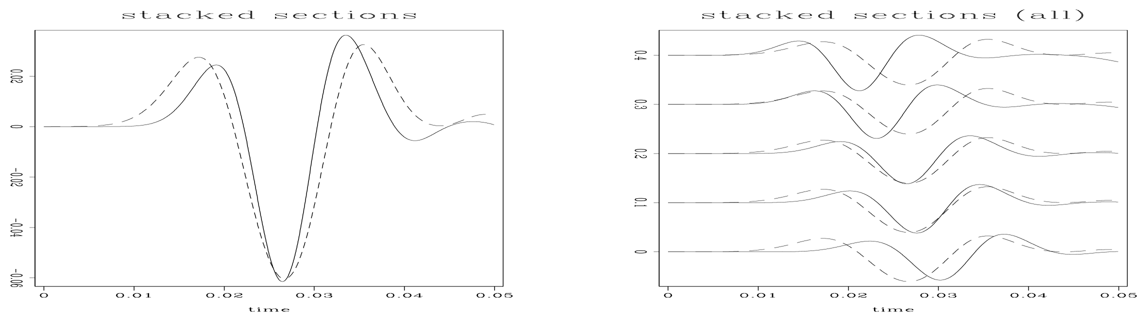

Figure 3: On the left we see the stacked sections of original (dashed line) and smoothed medium (solid line). The right picture shows (from bottom to top) the stacked section for the medium with averaged inverse squared velocity compared with the stacked section of the unsmoothed medium (dashed line), the stacked section for inverse velocity averaging, our method, velocity averaging and squared velocity averaging (this corresponds to averaging the stiffness parameters).

yield better results than the other methods for a smoothing radius smaller than $1/4$ of a wavelength.

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APPENDIX A

The following equations show the effective medium parameters in the low frequency limit for an elastic isotropic random medium.

In the case of 3D random media, we obtain :

$$\alpha_{eff} = \alpha \left[1 + \frac{1}{2} \frac{\lambda_o^2}{(\lambda_o + 2\mu_o)^2} \sigma_{\lambda\lambda}^2 + \frac{2}{3} \frac{\lambda_o \mu_o}{(\lambda_o + 2\mu_o)^2} \sigma_{\lambda\mu}^2 \right. \tag{A-1}$$

$$\left. + \frac{2}{5} \frac{\mu_o^2}{(\lambda_o + 2\mu_o)^2} \sigma_{\mu\mu}^2 + \frac{4}{15} \sigma_{\mu\mu}^2 \right]$$

$$\beta_{eff} = \beta \left[1 + \frac{1}{5} \sigma_{\mu\mu}^2 + \frac{2}{15} \frac{\alpha^2}{\beta^2} \sigma_{\mu\mu}^2 \right] \tag{A-2}$$

In these equations, α and β denote the wavenumbers of the homogeneous background medium, where density and elastic parameters have their mean values. α_{eff} and β_{eff} are the wavenumbers of the effective medium in the low frequency limit.
$\sigma_{\lambda\mu}^2$ is the normalized cross variance of the Lamé parameters λ and μ.

For 2D random media, the result reads :

$$\alpha_{eff} = \alpha \left[1 + \frac{1}{2} \frac{\lambda_o^2}{(\lambda_o + 2\mu_o)^2} \sigma_{\lambda\lambda}^2 + \frac{\lambda_o \mu_o}{(\lambda_o + 2\mu_o)^2} \sigma_{\lambda\mu}^2 \right. \tag{A-3}$$

$$\left. + \frac{3}{4} \frac{\mu_o^2}{(\lambda_o + 2\mu_o)^2} \sigma_{\mu\mu}^2 + \frac{1}{4} \sigma_{\mu\mu}^2 \right]$$

$$\beta_{eff} = \beta \left[1 + \frac{1}{4} \sigma_{\mu\mu}^2 + \frac{1}{4} \frac{\alpha^2}{\beta^2} \sigma_{\mu\mu}^2 \right] \tag{A-4}$$