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Normal moveout velocities in 3D arbitrary anisotropic media

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ABSTRACT

We present analytical expressions for normal-moveout velocities of reflections from a horizontal reflector for homogeneous models of general anisotropy. The formulae are valid for strong anisotropy and can be applied inside and outside of symmetry planes. They are expressed either in terms of first derivatives of ray (group) velocity vector or in terms of second derivatives of phase velocity. The results can be reduced to the particular cases of symmetry planes or to the case of weak anisotropy by using perturbation theory. Numerical examples for triclinic Lavoux limestone with phase velocity anisotropy of about 10% are presented. The exact NMO-formulae allow to investigate the accuracy of the weak anisotropy approximation of the NMO-velocity. The weak anisotropy approximation of NMO velocity performs less good than the weak anisotropy approximation of the phase velocity. For phase velocity variations in the order of 10% relative errors of the NMO velocities are about 1.5%. This appears to be sufficient for a velocity analysis.

Methods of seismic processing designed for isotropic media are no longer applicable if the media are anisotropic. Elastic anisotropy seriously distorts the results of processing of seismic data like velocity analysis, stacking, depth conversion, migration, etc. It is also well known that anisotropy influences the normal moveout velocity of a reflector (Krey and Helbig, 1956). Most work on normal moveout in the past was related to transversely isotropic media or to media of elliptical anisotropy. Recently (Tsvankin, 1995) derived a formula for the normal-moveout velocity for arbitrary anisotropy, however, this results are valid in symmetry planes. (Sayers, 1995) used spherical harmonics to describe reflection moveout in general anisotropic media and (Gajewski and Psenc̆k, 1997) derived a formula for moveout velocity for arbitrary weakly anisotropic media using perturbation theory. We will derive here analytical formulae of the normal-moveout velocity for arbitrary homogeneous anisotropic models which are valid for strong anisotropy and which are applicable outside of symmetry planes.

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NORMAL MOVEOUT IN ANISOTROPIC MEDIA

The expression normal moveout in this work relates to hyperbolic moveout, i.e., we obtain a strait line in a $t^2 - X^2$ graph and the normal moveout velocity corresponds to the slope of this line. As a starting point we use the expression of (Grechka and Tsvankin, 1996):

$$\begin{aligned} V_{\mathrm{NMO}}^{-2} &= \frac{\hat{T}}{2}\left[\frac{\partial \hat{p}_1}{\partial x_1}\cos^2\phi + \frac{\partial \hat{p}_2}{\partial x_2}\sin^2\phi \right. \\ &+ \left. \left(\frac{\partial \hat{p}_1}{\partial x_2} + \frac{\partial \hat{p}_2}{\partial x_1}\right)\cos\phi\sin\phi\right] \quad . \end{aligned} \tag{1}$$

Here p_i is the slowness vector, T is two-way travel time and t is one way travel time. ϕ specifies the orientation of the profile, x_i are mid point (CMP) coordinates, the hat above a quantity indicates the zero offset ray (here $x_1 \rightarrow 0$, $x_2 \rightarrow 0$). Eq. 1 can be used in ray modeling programs, since the partial derivatives of the slowness w.r.t. CMP coordinates can be obtained from results of dynamic ray tracing. In the following we show, that for the particular case of the zero offset ray we can derive analytical expressions for the derivatives occurring in eq. 1.

ALTERNATIVE EXPRESSIONS FOR THE NMO VELOCITY

Rays are most conveniently (and uniquely) described by ray coordinates γ_J, i.e.,

$$n_i = (\gamma_1, \gamma_2, \sqrt{1-\gamma_1^2-\gamma_2^2}) \quad , \tag{2}$$

where n_i corresponds to the phase normal of the ray. For the zero offset ray $\gamma_1 \rightarrow 0$ and $\gamma_2 \rightarrow 0$, i.e., $\hat{n}_i = (0,0,1)$. Since the slowness is defined as $p_i = n_i/v$, where n_i is the phase normal and v is the phase velocity along n_i, we obtain for the partial derivatives of p_i w.r.t. ray coordinates see, e.g., (Gajewski, 1993)

$$\frac{\partial p_i}{\partial \gamma_J} = \frac{1}{v}\left(\frac{\partial n_i}{\partial \gamma_J} - p_i w_k \frac{\partial n_k}{\partial \gamma_J}\right) \quad , \tag{3}$$

where w_i corresponds to the ray velocity vector (upper case indices take values 1 and 2). The partial derivatives of n_i are obtained from eq. 2. The remaining problem is to find the transformation matrix, which transforms the partial derivatives of p_I from ray to CMP coordinates. For a homogeneous anisotropic medium we obtain from the ray tracing system the following set of parametric equations for one-way travel time and CMP coordinates,

$$t(\gamma_J) = \frac{h}{w_3} \qquad x_I(\gamma_J) = h\frac{w_I}{w_3} \quad . \tag{4}$$

This expressions can be exploited to derive expressions for the sought transformation matrix $\frac{\partial \hat{\gamma}_I}{\partial x_J}$.

Assuming a horizontal reflector at depth h and using the parametric equations and the eikonal equation for anisotropic media one can show that the two-way traveltime of the zero offset ray is

$$\hat{T} = 2\hat{t} = \frac{2h}{\hat{v}} \quad , \tag{5}$$

where $\hat{v}$ is the phase velocity of the zero offset ray. Using eq. 4 we obtain a new expression for the normal moveout velocity,

$$\begin{aligned} V_{\mathrm{NMO}}^{-2} &= \frac{1}{\hat{D}\hat{v}}\left[\frac{\partial \hat{w}_2}{\partial \gamma_2}\cos^2\phi + \frac{\partial \hat{w}_1}{\partial \gamma_1}\sin^2\phi \right. \\ &- \left. 2\frac{\partial \hat{w}_1}{\partial \gamma_2}\cos\phi\sin\phi\right] \quad , \end{aligned} \tag{6}$$

with $\hat{D} = \frac{\partial \hat{w}_1}{\partial \gamma_1}\frac{\partial \hat{w}_2}{\partial \gamma_2} - \frac{\partial \hat{w}_1}{\partial \gamma_2}\frac{\partial \hat{w}_2}{\partial \gamma_1}$. Analytical expressions for the partial derivatives of the ray velocity vector w_i w.r.t. ray coordinates γ_J can be found in, e.g., (Gajewski and Psenc̆k, 1990).

The eikonal equation for anisotropic media $w_i p_i = 1$ allows us to derive another form of the NMO velocity in arbitrary anisotropic media. Differentiation of the eikonal equation two times w.r.t. ray coordinates links 2nd and 1st derivatives of ray velocity together, because we get in the limiting case of the zero offset ray

$$\frac{\partial^2 \hat{w}_3}{\partial \gamma_I \partial \gamma_J} = -\frac{\partial \hat{w}_I}{\partial \gamma_J} \quad . \tag{7}$$

Rewriting the eikonal equation into the form $w_i n_i = v$ and two times differentiation w.r.t. ray coordinates gives

$$\frac{\partial \hat{w}_I}{\partial \gamma_J} = \hat{v}\delta_{IJ} + \frac{\partial^2 \hat{v}}{\partial \gamma_I \partial \gamma_J} \quad , \tag{8}$$

where we have used eq. 7. Putting eq. 8 into 6 we can write an expression for the NMO velocity which depends entirely on the phase velocity and its derivatives:

$$\begin{aligned} V_{\mathrm{NMO}}^{-2} = \frac{1}{\hat{v}\hat{\mathcal{D}}}&\left[\left(\hat{v} + \frac{\partial^2 \hat{v}}{\partial \gamma_2^2}\right)\cos^2\phi \right. \\ + \left(\hat{v} + \frac{\partial^2 \hat{v}}{\partial \gamma_1^2}\right)\sin^2\phi &- \left. 2\frac{\partial^2 \hat{v}}{\partial \gamma_1 \partial \gamma_2}\sin\phi\cos\phi\right] \end{aligned} \tag{9}$$

where $\hat{\mathcal{D}} = \left(\hat{v} + \frac{\partial^2 \hat{v}}{\partial \gamma_1^2}\right)\left(\hat{v} + \frac{\partial^2 \hat{v}}{\partial \gamma_2^2}\right) - \left(\frac{\partial^2 \hat{v}}{\partial \gamma_1 \partial \gamma_2}\right)^2$. With eqs. 1, 6 and 9 we have a set of NMO equations for arbitrary anisotropic media, depending either on the derivatives of the slowness vector w.r.t. CMP coordinates or to the 1st derivatives of the ray velocity vector or the 2nd derivatives of the phase velocity w.r.t. ray coordinates. The expressions can be chosen in accordance to the problem at hand, i.e., which data are available.

NMO VELOCITY IN SYMMETRY PLANES

The last equation (or alternatively eq. 6) allows to obtain an expression for the NMO-velocity in symmetry planes. This is a 2-D problem since the rays are located in the plane of incidence. Let us consider the $x_1 - x_3$ plane. Here the NMO equations 6 and 9 result in the following expression

$$V_{\mathrm{NMO}}^2 = \hat{v}\frac{\partial \hat{w}_1}{\partial \gamma_1} = \hat{v}^2 \left(1 + \frac{1}{\hat{v}}\frac{\partial^2 \hat{v}}{\partial \gamma_1^2}\right) \quad . \tag{10}$$

This result corresponds to (Tsvankin, 1995) for the NMO-velocity in symmetry planes of arbitrary anisotropic media with a dipping interface if specified for a horizontal reflector (see his eq. 9). Expressions for the other symmetry planes of the medium are obtained accordingly.

SPECIFICATION FOR WEAK ANISOTROPY

In this section we consider the moveout of quasi P-waves. For weak anisotropy we assume an isotropic background medium with isotropic constant P-wave velocity α. The anisotropy of the medium is considered to be weak. The phase velocity consists of two parts: the isotropic background velocity α and a small correction Δv, which represents the influence of the anisotropy, i.e., $v = \alpha + \Delta v$. This correction for quasi P-waves is (Cervený & Jech, 1982) $\Delta v = S/2\alpha^2$ and

$$S = a_{ijkl} n_i n_j n_k n_l - \alpha^2 \tag{11}$$

Here a_{ijkl} are density normalized elastic coefficients. Two times differentiation of the last expression, considering the zero offset ray and putting the results into eq. 9 we obtain (by keeping only linear terms) another version of the NMO equation which is approximately valid for weakly anisotropic media

$$\begin{aligned} V_{\mathrm{NMO}}^{-2} \quad \approx \quad & \frac{1}{\alpha^2}\left[1 - \frac{\hat{S}}{\alpha^2} - \frac{1}{2\alpha^2}\left(\frac{\partial^2 \hat{S}}{\partial \gamma_1^2}\cos^2\phi \right.\right. \\ & \left.\left. + \frac{\partial^2 \hat{S}}{\partial \gamma_2^2}\sin^2\phi + 2\frac{\partial^2 \hat{S}}{\partial \gamma_1 \partial \gamma_2}\sin\phi\cos\phi\right)\right] \quad . \end{aligned} \tag{12}$$

Since the derivatives of S are easily obtained from eq. 11 and using the vertical velocity as background velocity, i.e., $v_\perp^2 = a_{33}$ (compressed notation for elastic coefficients), we arrive at the final expression

$$\begin{aligned} V_{\mathrm{NMO}}^{-2} \quad = \quad & v_\perp^{-2}(1 - 2\delta^{(2)}\cos^2\phi - 2\delta^{(1)}\sin^2\phi \\ - \quad & 4\bar{\delta}\sin\phi\cos\phi) \quad , \end{aligned} \tag{13}$$

with the anisotropy parameters $\delta^{(1)} = \frac{a_{13}+2a_{55}-a_{33}}{a_{33}}$, $\delta^{(2)} = \frac{a_{23}+2a_{44}-a_{33}}{a_{33}}$ and $\bar{\delta} = \frac{a_{36}+2a_{45}}{a_{33}}$. These anisotropy parameters are formally similar to Thomsen's (1986) δ parameter for transversely isotropic media, however, they are expressed in linear form and generalized for arbitrary anisotropy (for more details on anisotropy parameters for arbitrary anisotropic media, see (Gajewski and Pšenčík, 1996; Mensch and Rasolofosaon, 1997; Tsvankin, 1997).

In the paragraphs above several variants of equations for the hyperbolic NMO velocity in homogeneous media with a horizontal reflector were derived. They are applicable within and outside of symmetry planes for arbitrary anisotropy. In the next section we will apply these formulae to a triclinic model and discuss the accuracy of the weak anisotropy approximation.

NUMERICAL EXAMPLES

A saturated Lavoux limestone is considered (for elastic parameters, see Mensch and Rasolofosaon, 1997. The phase velocity variations of this sample in dependence of polar angle and azimuth are displayed in Fig.1. The maximum qP wave phase velocity variation is a little more than 10%. The governing anisotropy parameters of the NMO equation 13 are $\delta^{(2)} = -0.1$, $\delta^{(1)} = -0.08$ and $\bar{\delta} = 0$, i.e., they are in the order of 10%. In Fig. 2

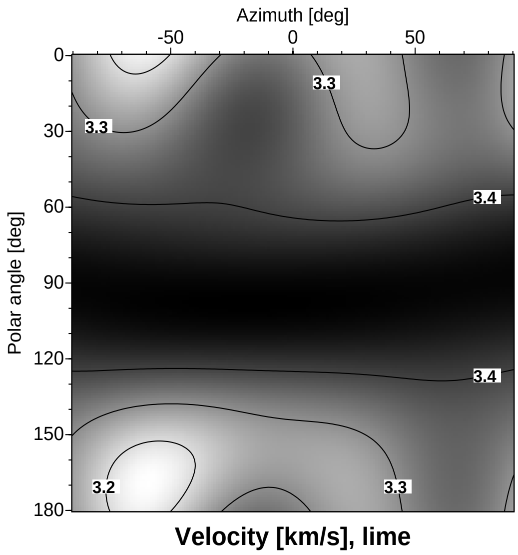

Figure 1: qP wave phase velocities of saturated Lavoux limestone. Maximum phase velocity variations are about 10%.

the exact NMO velocity (black curve) and the NMO velocity in weak anisotropy approximation (WA) is shown. The WA value is always larger than the exact value, which was computed by eq. 1 using the ANRAY program package (Gajewski and Pšenčík, 1990) to compute the partial derivatives of the slowness vector by dynamic ray tracing. A slight shift of minimum and maximum between exact and WA solution is visible in Fig.2 which would lead to an error in the orientation of the anisotropic reference coordinate system

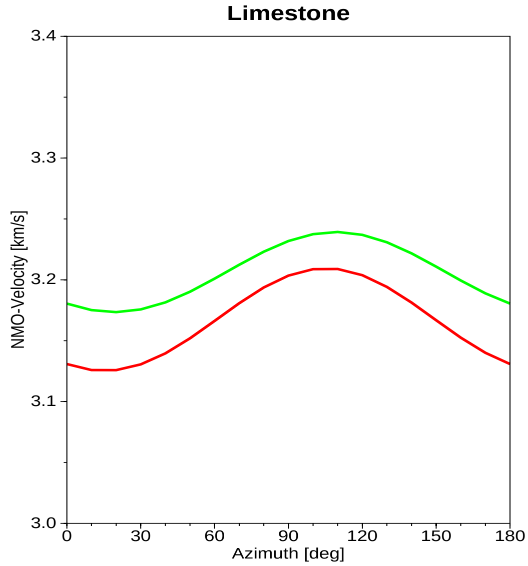

Figure 2: qP wave NMO velocity of saturated Lavoux limestone, exact (black curve) and for the weak anisotropy approximation (gray curve).

(e.g. the orientation of an inclined fracture system). However, this deviations are fairly small. The relative error between exact and WA result is shown in Fig. 3. The maximum relative error is about 1.6% and the smallest error is about 0.8%. This is less good than the weak anisotropy approximation of the phase velocity, which is everywhere below 0.5% for this sample. This, however, is no surprise, since the linearization of the NMO velocities required the neglect of more higher order terms (see above), than linearizing the phase velocity.

CONCLUSIONS

Exact formulae for moveout velocities in arbitrary homogeneous anisotropic media were derived. The formulae are valid as long as the $T^2 - X^2$ curve can be approximated by a straight line (i.e., a hyperbolic moveout is assumed). The extension of the obtained results to layered media resulting in a Dix-style type of formula should be straight forward. Also the extension to dipping interfaces should not involve major complications, since this only changes the transformation from ray to CMP coordinates. All other relations are gained through a rotation, such that the zero offset ray hits the inclined interface perpendicularly.

The exact NMO equations allowed to investigate the accuracy of the weak anisotropy approximation within and outside of symmetry planes. It is generally less precise than the weak anisotropy approximation of the phase velocity for the same model. This, however, is no surprise, since the weak anisotropy approximation of the NMO velocity accumu-

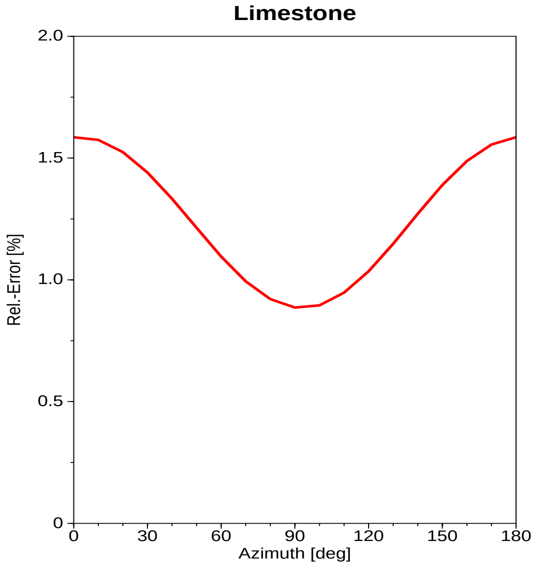

Figure 3: Relative error between exact and approximate NMO velocities for saturated Lavoux limestone.

lates more errors owing to the neglect of several higher order terms. However, if the magnitude of the governing anisotropy parameters is about 10%, the weak anisotropy approximation has maximum errors of about 1.5%, which is sufficient for a velocity determination procedure.

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PUBLICATIONS

More details on the derivation of the NMO velocity for the weak anisotropy approximation are described in (Gajewski and Psenc´k, 1997).