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Finite-difference traveltime computations for anisotropic media

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ABSTRACT

A 2D finite-difference eikonal solver for elliptically anisotropic media is developed. It is based on a second-order approximation of traveltimes; points are successively timed on expanding squares around the source. Since elliptical anisotropy is of limited significance for real subsurfaces a perturbation scheme of first order is introduced to consider, in principle, arbitrary symmetry systems. Using reference media with elliptical anisotropy improves accuracy of this FD-perturbation method compared to usage of isotropic reference media.

INTRODUCTION

Several finite-difference (FD) methods for computing traveltimes to a large number of points of a discretized subsurface isotropic model have been developed, e.g. (Vidale, 1988), and some extensions to anisotropic media also exist, e.g. (Dellinger, 1991; Lecomte, 1993). The latter are either restricted to transversely isotropic media or involve the unstable process of solving a higher order polynomial numerically. We consider a different approach for anisotropic media. Since anisotropy in the earth is usually weak ($< 10\%$) we compute traveltimes in slightly anisotropic media by perturbation. A highly efficient method using reference isotropic models where the perturbation integrals are introduced into the FD-eikonal solver by (Vidale, 1988) was implemented by (Ettrich and Gajewski, 1996). We now improve accuracy by extending Vidale's eikonal solver and the FD-perturbation method to reference media with elliptical anisotropy.

REFERENCE MODELS OF ELLIPTICAL ANISOTROPY

P-wave slowness surfaces are elliptically shaped, if elastic parameters of transversely isotropic media follow the relation $(a_{1111} - a_{2323})(a_{3333} - a_{2323}) = (a_{1133} + a_{2323})^2$. Then the slowness surface (in 2D) reads

$$Ap_x^2 + Cp_xp_z + Bp_z^2 = 1. \tag{1}$$

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p_x, p_z denote components of the slowness vector, and A, B, and C are functions of vertical and horizontal phase velocity of the considered wave type and of the orientation of the crystal coordinate system. The procedure for the derivation of the reference traveltime t_3

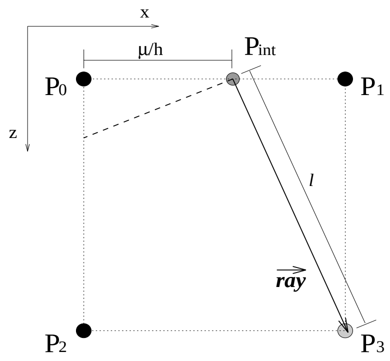

Figure 1: Grid cell with known traveltimes at P_0, P_1 and P_2. A locally plane wavefront is assumed to propagate through the cell to compute t_3 at P_3.

at P_3 (see Figure 1) is analogous to the isotropic case (Vidale, 1988). In equation (1) the slowness vector is approximated by centered finite differences, and we obtain

$$t_3 = t_0 + \frac{(B-A)(t_1-t_2) \pm \sqrt{4h^2(A+B+C) - (t_1-t_2)^2(4AB-C^2)}}{A+C+B}, \tag{2}$$

where t_0,t_1, and t_2 are traveltimes at P_0, P_1, and P_2, respectively. As in (Vidale, 1988) the computational scheme follows squares expanding around the source. Causality requires that traveltimes at those points of the current square are computed first where the component of the group velocity vector which is tangent to the actual side of the square changes its sign. In isotropic media this is equivalent to consider the phase velocity vector by choosing the minimum traveltime point, however, for elliptical anisotropy this requires to compute the angle α_{G} of the group velocity vector:

$$\alpha_{\text{G}} = \arctan\left(\frac{C + 2A\tan\alpha_{\text{Ph}}}{C\tan\alpha_{\text{Ph}} + 2B}\right), \tag{3}$$

where α_{Ph} is the angle of the slowness vector, which is known by differences of traveltimes.

An application of the method is demonstrated in Figure 2. For stability and for reduction of staircase artifacts due to dipping interfaces a slight smoothing was necessary.

Considering any other anisotropic medium, to first-order the traveltime t between points S and P is obtained by adding an integral expression to the traveltime $t_{\text{ellip}}(P,S)$ in the elliptical reference medium:

$$t(P,S) = t_{\text{ellip}}(P,S) - \frac{1}{2}\int\limits_{\text{ray}_{(P,S)}} \Delta a_{ijkl} g_j g_k p_i p_l \, dt. \tag{4}$$

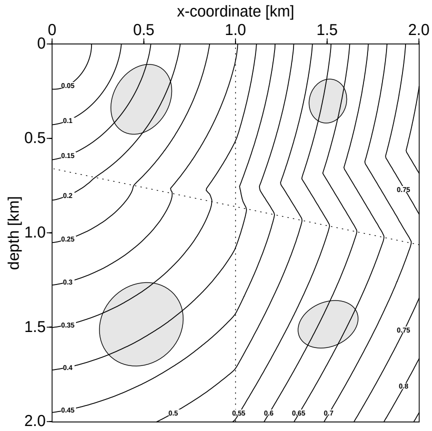

Figure 2: Wavefronts in an elliptically anisotropic model. Dashed lines display interfaces before smoothing. Phase velocities of the model blocks are displayed by ellipses. Size and rotation of ellipses reflect velocity ratios.

All quantities of the integrand, i.e., slowness vector p_i, polarization vector g_i and differences of elastic coefficients Δa_{ijkl} between the given anisotropic and the elliptically anisotropic reference medium have to be computed along the ray in the reference medium. The raypath in the background medium is approximated by ray segments corresponding to the plane waves in each cell (see Figure 1) (Ettrich and Gajewski, 1996).

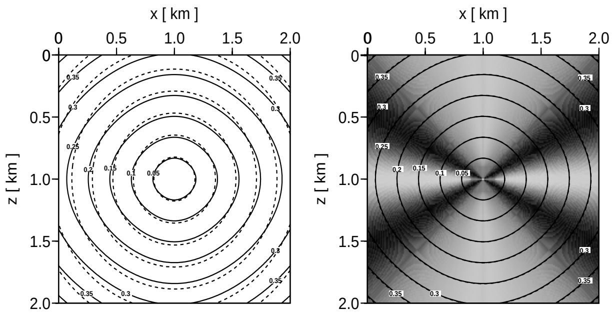

Figure 3: Left figure: Exact wavefronts (solid) in a homogeneous model of transversely isotropic Taylor sandstone and wavefronts (dashed) in the isotropic reference medium. Right figure: Exact (solid) and FD-perturbation wavefronts (dashed) with an underlying gray scale image of relative errors. Black color corresponds to a maximum error of 0.76%.

To demonstrate the advantage of reference media of elliptical anisotropy we show results for a homogeneous model of Taylor sandstone (Thomsen, 1993) with Thomsen parameters $\epsilon = 0.11$ and $\delta = -0.035$. Accuracy is considerably higher in Figure 4 for an elliptically anisotropic reference medium than in Figure 3 for an isotropic reference medium.

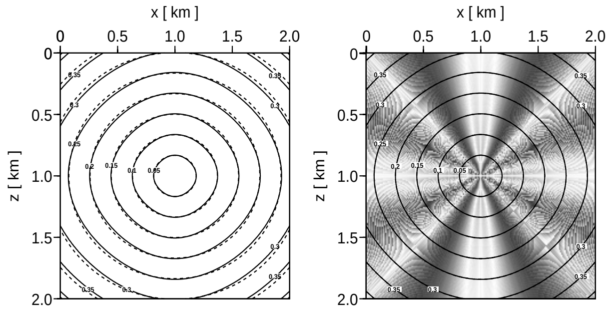

Figure 4: Same as Figure 3 but with elliptically anisotropic reference medium. Black color corresponds to a maximum error of 0.2%.

CONCLUSION

The extension of Vidale's FD-eikonal solver to media with elliptical anisotropy turns out to be stable in weakly smoothed models. Application of perturbation techniques allows to consider arbitrary symmetry systems. However, the method must first be extended to 3D. Isotropic or elliptically anisotropic reference media can be used depending on the higher importance of either speed or accuracy of the computation.

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PUBLICATIONS

Detailed results will be published in (Ettrich and Gajewski, 1997).