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Seismic characterization of statistical properties of fractured composite materials

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ABSTRACT

A wavefield propagating in a scattering medium can be split up into coherent and incoherent parts, whose intensity changes depending on traveldistance and frequency. Looking at the statistics of the transmitted wavefield, we have found a quantity that allows to characterize a medium and estimate its statistical parameters. Laboratory measurements were carried out on FRC (fibre reinforced composite) specimens serving as a model for heterogeneous rocks with cracks. The underlying theory (Rytov approximation) neglects backscattering and assumes wavelengths shorter than the characteristic length of inhomogeneities.

We introduce the parameter $\langle \epsilon^2 \rangle$ being a measure for the ratio of incoherent to coherent wavefield intensity. Applying these concepts to real data obtained from ultrasonic measurements, theoretical predictions for the relation between $\langle \epsilon^2 \rangle$ and frequency are confirmed. One expects a quadratic power law, resulting in a straight line in a double-logarithmic diagram. This suggests scattering at large-scale inhomogeneities rather than at small individual structures (cracks). The evaluation of P-wave data from upper crust events shows a similar behaviour. From the calculation of $\langle \epsilon^2 \rangle$, it is possible to infer medium properties, such as the variance and the correlation length of medium parameters.

As a result, we have found a significant and robust parameter extracted from a wavefield that allows a qualitative and quantitative statistical characterization of the medium penetrated by the wave.

INTRODUCTION

Seismic signals are distorted by medium inhomogeneities due to, among other effects, scattering. This is a commonly observed fact in various seismological applications, such as teleseismics or, for a different frequency range, non-destructive testing. Scattering mechanisms are inherently dependent on the frequency of the wave, medium contrasts and the size of inhomogeneities. What we desire is an inversion based on wavefield characteristics that yields information on medium properties, i. e. the size and kind of

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scattering objects. The effect of random inhomogeneities on the phase velocity has been studied by (Shapiro et al., 1996). A treatise on wave attenuation caused by scattering can be found in (Shapiro and Kneib, 1993).
We present a new concept by defining a statistical parameter gained from the wavefield that relates incoherent to coherent intensity. Based on the Rytov approximation derived by (Ishimaru, 1978), a random acoustic medium is assumed, with parameters fluctuating weakly and large-scale inhomogeneities compared to the wavelength.
Considering meanfield theory (Ishimaru, 1978), we then verify the significance of the introduced statistical parameter with respect to the medium.

RANDOM MEDIA

As the media we deal with are chosen to be random, a brief introduction to the concept and the mathematical description of random media follows.
Physical quantities that describe a medium, such as density and the Lamé parameters λ and μ, can be conceived as stationary random fields in space $f(\vec{r})$. These are characterized by their statistical moments:

- average value:

$$\langle f(\vec{r})\rangle \equiv \int_{-\infty}^{\infty} f P_1(f)\, df$$

  (first moment)

- second moment:

$$\langle f_1 f_2\rangle \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1 f_2 P_2(f_1, f_2)\, df_1 df_2$$

- variance:

$$\sigma^2 = \langle (f - \langle f\rangle)^2\rangle\ .$$

$P_i(f_i)$ denotes probability density of realization f_i at $\vec{r}$ and i the realization index. The angular brackets denote statistical ensemble averaging.
A statistical ensemble contains a set of realizations with identical moments. If the moments of an ensemble are equal to the moments of the realizations for a given argument, i. e. space, the medium described by the random field is called an ergodic medium.
The autocorrelation function can be written as

$$B_f = \langle f(\vec{r} + \vec{dr}) - \langle f\rangle)(f^*(\vec{r}) - \langle f\rangle)\rangle$$

and equals the second moment for $\langle f \rangle \equiv 0$.

Common autocorrelation functions that describe a random medium are the Gaussian, the exponential and the von-Karman function. The correlation length, usually denoted by a, follows from the choice of autocorrelation function and is proportional to the size of inhomogeneities. It is a measure of how strongly the parameter varies in space.
A complete treatment of random process theory can be found in (S. M. Rytov and Tatarskii, 1987).

WAVE PROPAGATION IN RANDOM MEDIA

At a point $\vec{r}$ in a random medium, the wavefield can be described as

$$u(\vec{r},t) = \langle u(\vec{r},t)\rangle + u_f(\vec{r},t) \; . \tag{1}$$

$\langle u \rangle$ represents the coherent field (meanfield) and u_f is the fluctuation of u and called the incoherent field. It is $\langle u_f \rangle = 0$.
We also define coherent, incoherent and total intensity as

$$\begin{aligned} I_c &= |\langle u \rangle|^2 \\ I_f &= \langle |u_f|^2 \rangle \\ I_t &= I_c + I_f \; . \end{aligned}$$

In the validity range of the Rytov approximation ((Ishimaru, 1978)), we consider an acoustic wavefield with neglected backscattering. This implies constant total intensity ($I_t = const.$).
We now introduce the fluctuation parameter ϵ by

$$\epsilon \equiv \frac{|u - \langle u \rangle|}{|\langle u \rangle|} = \frac{|u_f|}{|\langle u \rangle|} \tag{2}$$

or alternatively, in terms of intensities

$$\langle \epsilon^2 \rangle = \frac{I_f}{I_c} \; . \tag{3}$$

$\langle \epsilon^2 \rangle$ represents a measure for the wavefield fluctuation. It depends on frequency, as higher frequencies are generally scattered more strongly than lower ones, and it is subject of interest what kind of scattering mechanism we have in the medium. Of course, $\langle \epsilon^2 \rangle$

also depends on medium parameters, such as variance σ^2 and correlation length a, as will be shown in the following.
The region where $\langle\epsilon^2\rangle \ll 1$ is called the weak fluctuation region; for $\langle\epsilon^2\rangle \gg 1$, the incoherent intensity dominates and the wave propagates in a region of strong fluctuations ((Shapiro and Kneib, 1993)).
From (3), it follows for $\langle\epsilon^2\rangle \ll 1$

$$\langle\epsilon^2\rangle \approx 2\alpha_{\langle u\rangle} L \tag{4}$$

with $\alpha_{\langle u\rangle}$ being the meanfield scattering coefficient. For $\alpha_{\langle u\rangle}$, one obtains for harmonic waves and by using the Born approximation ((Shapiro and Kneib, 1993))

$$\alpha_{\langle u\rangle} \sim \sigma^2 a k^2 \ . \tag{5}$$

Hence, combining (4) and (5), we get

$$\langle\epsilon^2\rangle = 2\sigma^2 a(\frac{2\pi}{c}\nu)^2 L \tag{6}$$

where ν denotes frequency and L traveldistance of the wave. Taking the logarithm, a simple linear expression follows

$$\ln\langle\epsilon^2\rangle = \ln(2\sigma^2 a(\frac{2\pi}{c}\nu_o)^2 L) + 2\ \ln(\frac{\nu}{\nu_o}) \tag{7}$$

which relates $\langle\epsilon^2\rangle$ and ν in an easily verifiable way ($\nu_o = 1$ Hz).

APPLICATION TO REAL DATA

In the field of non-destructive testing, ultrasonic measurements represent a versatile method for the investigation of materials with respect to their elastic properties. In this work, real data from such an experiment serve as test input for the theory. The measurement is carried out on a fibre-glass reinforced composite sample (see Fig. 1) that finds widespread use in engineering and construction applications. Fig. 2 shows a section comparable with a seismic zero-offset section.

The experimental goal is to characterize and distinguish the sample in terms of increasing degree of damage. Firstly, the measurement is carried out on the undamaged sample with an induced mean frequency of 10 MHz.

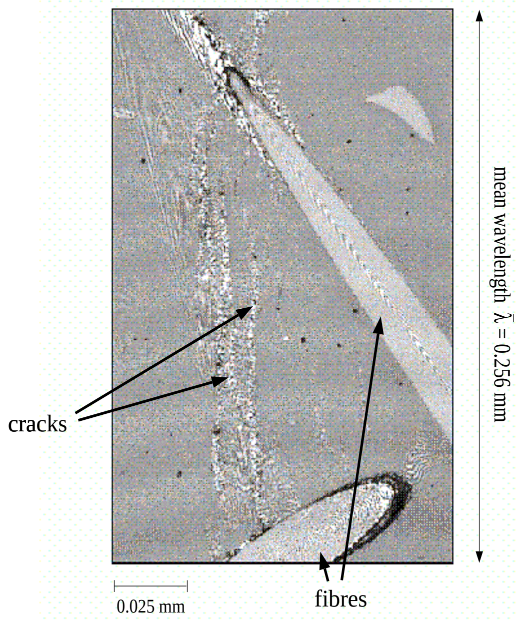

Figure 1: Enlarged photograph of a fibre-reinforced composite, a model for a rock with cracks.

Then, a strain of 1% is applied perpendicular to the wave propagation direction. Afterwards, the measurement is made on the released (presumably internally damaged) sample, and the procedure gets repeated for strains of 2% and 3% .
The externally applied strain induces cracks within the sample, as displayed in Fig. 1, and it is reasonable to assume that the number of cracks increases with strain. Microscopical examinations confirm that the crack width does not exceed the order of 10^{-5} m, as already suggested by Fig. 1. The mean wavelength of the signal is $\bar{\lambda} = 2.56 \cdot 10^{-4}$ m with $c = 2.56$ km/s .
A good coupling between source (ultrasonic piezo transducer) and sample is guaranteed by putting the sample in water during the measurement. For the computation of $\langle\epsilon^2\rangle$, we use the transmitted signal (one-way through the sample) for data quality reasons rather than the reflected signal shown in the lower half of Fig. 2.

RESULTS FOR THE FLUCTUATION PARAMETER $\langle\epsilon^2\rangle$

We now utilize the supplied real data sets as input for the theoretical considerations made above. We choose the transmitted signal and compute $\langle\epsilon^2\rangle$ by the governing equation (3). The $\langle\epsilon^2\rangle$ - frequency relation is being evaluated and discussed as follows:

Fig. 3 shows the overall dependence between ln$\langle\epsilon^2\rangle$ and the frequency ν for experiments with different strains having been applied. The frequency ranges from 0 to 16 MHz, comprising the weak fluctuation region. For higher frequencies, the wavefield fluctuation tends to a constant at high level (saturation occurs), which is not subject of

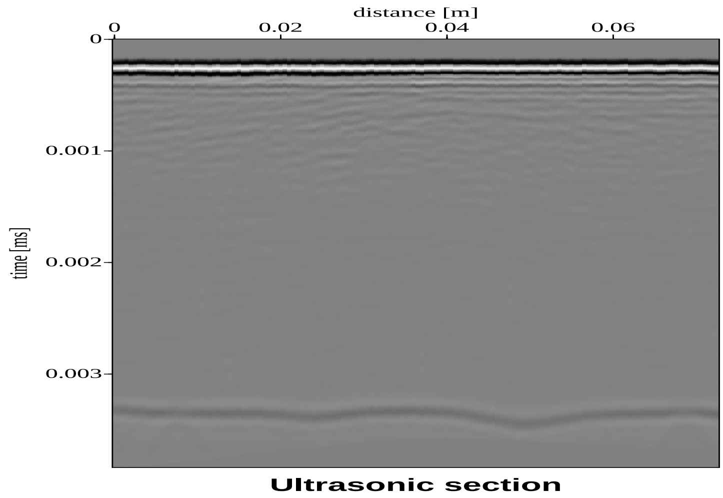

Figure 2: The early amplitude represents the input signal; the signal in the lower half of the section is the reflection at the bottom of the sample.

further interest here.
Qualitatively, a clear distinction can be made between different states of the sample, depending on strain. In the range of 3.5 MHz to 12 MHz, the curves display a fairly smooth behaviour. For lower frequencies ($<$ 3 MHz), some high $\langle\epsilon^2\rangle$ amplitudes occur, but they can be interpreted as artifacts due to the frequency content of the input signal.

Taking a closer look at the smooth part of the curves, Fig. 4 shows the $\ln\langle\epsilon^2\rangle - \ln\nu$ dependence for 1% strain. According to equation (7), one expects a linear relation, i. e. a straight line with a slope of 2, if the assumptions made on the medium are valid. In Fig. 4, the curve, if approximated by a straight line, shows a slope very close to the predicted value of 2. This is also the case for other strains and even for experiments with induced mean frequencies other than 10 MHz carried out on an identical sample.
It is important to note that no ν^4-dependence (Rayleigh scattering) of the fluctuation is observed.
Again recalling equation (7), quantitative interpretations about the medium are feasible, which is an important motivation for this work. For a slope of 2 in Fig. 4 and given $((\ln\langle\epsilon^2\rangle)_o, (\ln(\frac{\nu}{\nu_o}))_o)$, the explicit calculation of $\sigma^2 a$ is possible.

CONCLUSIONS

We have based our proceeding on the Rytov approximation for wavefields in random media. This involves large-scale inhomogeneities ($a > \lambda$) and smooth parameter variation. If, in our example, scattering happened at individual cracks, we should observe Rayleigh

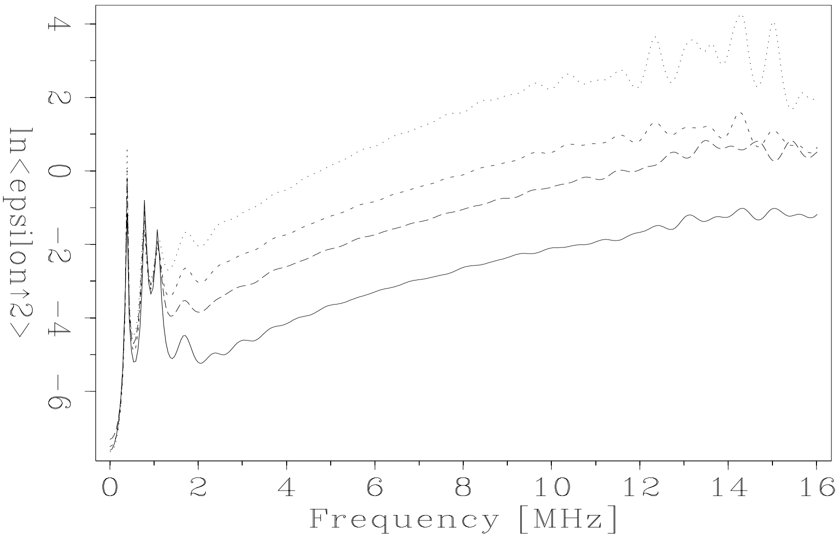

Figure 3: $\ln\langle\epsilon^2\rangle$ - frequency dependence for different strains applied: 0% (solid), 1% (large dashed), 2% (fine dashed), 3% (dotted).

scattering with a ν^4 dependence, as $\lambda \gg a$. This is not confirmed by the results. On the contrary, the Rytov approximation works remarkably well. As a consequence, scattering must occur on large-scale objects. This observation gives rise to the assumption that regions of increased crack density that are themselves fairly homogeneous and large compared to the wavelength cause the scattering of the wave. Since the wavefield fluctuation only depends on strain, other scattering mechanisms can certainly be excluded. The size of those regions is determined by the evaluation of $\sigma^2 a$.
Similar investigations using teleseismic data have been made by (Ritter et al., 1997) in order to ascertain statistical inhomogeneities of the lithosphere.
To conclude, we have found a significant and robust parameter derived from the wavefield that allows to distinguish media with different scattering properties and to characterize media quantitatively by the computation of $\sigma^2 a$.

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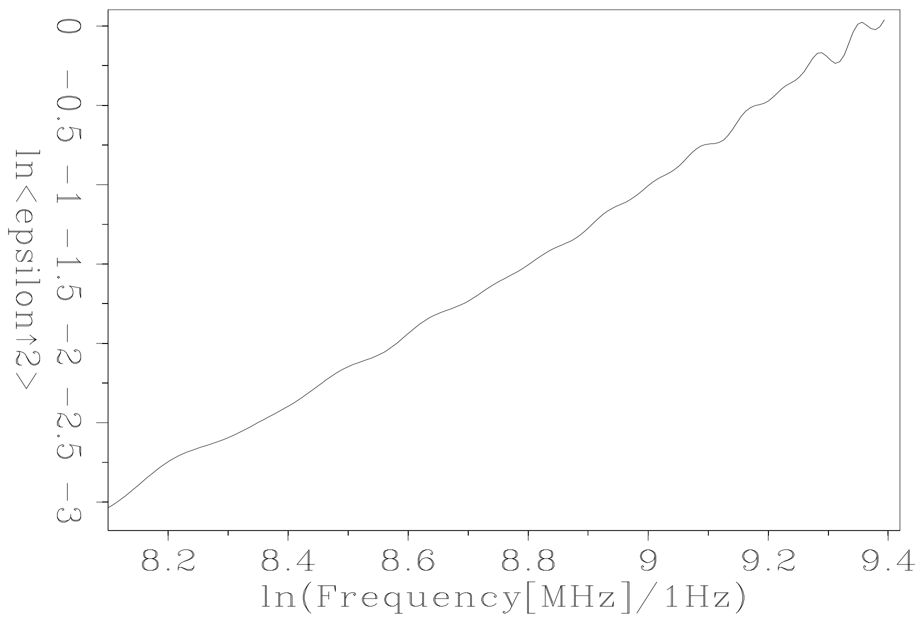

Figure 4: $\ln\langle\epsilon^2\rangle - \ln\nu$ dependence for 1% strain; note the asymptotic behaviour toward a straight line.

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